THE
A B C OF ALGEBRA

P. B. BALLARD, M.A., D.Lit.

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BY

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PREFACE

One of the aims of this book is to make Algebra as clear and real to the pupil as Arithmetic. Algebra indeed is exhibited as Arithmetic written in a different language, a language which, though it refers to the same basic principles, extends those principles further into the world of number and renders them applicable to a wider range of facts. The pupil is therefore constantly required to relate the Algebra which he is learning to the Arithmetic, Mensuration, and Geometry which he has already learnt.

Directed numbers are always a stumbling-block to the beginner. The notion of the plus and minus signs being used as adjectives and not as verbs is new to the mere arithmetician, and the rule of signs becomes a rich source of bewilderment. It is hoped that some of the difficulties are removed in this book, and that the rule of signs is rendered more intelligible by the treatment it receives in Exercise 27.

Some repetition will be noticeable. This repetition is intentional, for it serves two valuable purposes: it allows greater prominence to be given to important principles than to unimportant, and it affords abundant practice in those matters where the pupil is peculiarly liable to come to grief. Brackets preceded by a minus sign, for instance, occur over and over again. In fact, the requirements of sound learning are allowed to overrule the claims of economical statement and logical arrangement.

It is not pretended that this book will enable the pupil to dispense with the teacher's help, but it is claimed that the book is largely self-explanatory, and lends itself to a large measure of individual and independent study.

In furtherance of this purpose it is recommended that the introduction to the exercises be studied by the pupil as *actively* as his present knowledge will permit. When possible he should anticipate the text. When, for instance, a model example is worked in the book, the pupil, rather than follow the steps in the book, should work the example independently and then compare his own working with the author's.

A word about nomenclature. The abstract number five can be presented in three ways: five, 5, and x. The child calls the first a word, the second a figure, and the third a letter; and this clear and simple terminology is adopted in this book. It is free from the fog which surrounds such words as symbol, digit, and quantity.

The subject-matter goes as far as quadratic equations, but does not include them. Though the book is specially designed for senior, central, and preparatory schools, it provides an introductory course for beginners of all kinds.

My thanks are due to Miss Nest Jones, B.A., for her friendly help in checking the answers.

P. B. B.

CHISWICK, 1938

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EXERCISE 1.—LETTERS INSTEAD OF FIGURES

Instead of 7 men, 7 tons, 7 miles, 7 days, etc., I can write 7. That is arithmetic.

Instead of 1, 2, 3, 4, 5, 6, etc., I can write n, or a, or x, or any other letter. That is algebra. Just as "6" is shorthand for "six," so are the letters of algebra a sort of shorthand for figures and words.

Instead of saying that the area of a rectangle is to be found by multiplying the number of units of length by the number of units of breadth I can write $A = l \times b$, or, better still, $A = lb$. That again is algebra.

Instead of writing $3 + 7$, $8 + 4$, $2 + 9$, $1 + 5$, etc., I can write $a + b$, or $m + n$, or $x + y$, which is algebraic shorthand for "the sum of two numbers."

Instead of writing all the numbers I can think of which are of the form $6 - 2$, $8 - 5$, etc., I can write $a - b$, or $m - n$, or $x - y$, which is shorthand for "the difference of two numbers."

In the same way $a \times b$, or $m \times n$, or $x \times y$—or, better still, ab, or mn, or xy—means "the product of two numbers."

Also, $a \div b$, or $a : b$, or $\frac{a}{b}$, is shorthand for "the ratio of two numbers," or "the quotient obtained by dividing one number by another."

In arithmetic, when numbers are placed side by side, with no sign between them, it means that they have to be added.

$$2\tfrac{1}{2} \text{ means } 2 + \tfrac{1}{2}$$
$$368 \text{ means } 300 + 60 + 8$$
$$\cdot 75 \text{ means } \frac{7}{10} + \frac{5}{100}$$

In algebra, however, when numbers are placed side by side they have to be multiplied.

$$ab = a \times b$$
$$xyz = x \times y \times z$$
$$(a + b)(a - b) = (a + b) \times (a - b)$$

$(a + b)(a - b)$ is algebraic shorthand for "the sum of two numbers multiplied by their difference," or, better still, "the product of the sum and difference of two numbers." The brackets show that the numbers inside have to be taken as one. The brackets weld them together.

$\dfrac{a+b}{a-b}$ is algebraic shorthand for "the sum of two numbers divided by their difference." Note that the line, or *vinculum*, is the same line as in $\div$, and the same as in $\dfrac{1}{2}$. No brackets are necessary, because the vinculum itself serves the same purpose as a pair of brackets.

Without using + or − express the following arithmetically:

1. $50 + 6$. 2. $100 + 80 + 7$. 3. $4 + \frac{3}{4}$. 4. $7 + \frac{5}{10}$.
5. $800 + 20 + 7 + \frac{3}{10} + \frac{6}{100}$.

Without using $\times$ or $\div$ or : express the following algebraically:

6. $m \times n$. 7. $p \times q \times r$. 8. $x \times (y + z)$. 9. $(x + y) \times z$.
10. $(p + r) \times (p - r)$. 11. $x \div y$. 12. $y : z$.
13. $m \div (m + n)$. 14. $(m + n) \div p$. 15. $(s + t) \div (s - t)$.
16. Use two letters and a plus sign to express all numbers of this form : $2 + 5$; $6 + 2$; $8 + 1$; $4 + 3$; etc.
17. Use two letters and a minus sign to express all numbers of this form : $5 - 2$; $8 - 5$; $6 - 1$; etc.
18. Use three letters, a plus sign, and a minus sign to express all numbers of this kind : $5 + 4 - 3$; $6 + 9 - 5$; $8 + 3 - 2$; $1 + 7 - 3$; etc.
19. Without using any sign express in the letters of algebra every number of this sort : $3 \times 2 \times 7$; $5 \times 4 \times 9$; $3 \times 8 \times 6$; etc.
20. Without using the sign for division express algebraically all numbers of this sort : $3 \div 2$; $8 \div 3$; $2 \div 7$; $5 \div 6$; etc.
21. Express your answer to question 17 entirely in words.
22. Express your answer to question 19 entirely in words.

EXERCISE 2.—ALGEBRAIC SHORTHAND

Write in the shorthand of algebra without using $\times$ or $\div$ or : the following expressions. Unless other letters are named use a, b, or c, preferably a and b.

1. Two numbers added together.
2. The sum of three numbers.
3. One number subtracted from another number.
4. The difference of two numbers.
5. The product of three numbers.
6. The sum of $(m + n)$ and $(m - n)$.
7. The difference of $(m + n)$ and $(m - n)$.
8. The product of $(m + n)$ and $(m - n)$.
9. The ratio of $(m + n)$ to $(m - n)$.
10. The sum of two numbers added to their difference.
11. The sum of two numbers plus their difference.
12. The sum of two numbers minus their difference.
13. The difference of the sum and difference of two numbers.
14. The product of the sum and difference of two numbers.
15. The sum of two numbers divided by their product.
16. The product of two numbers divided by their difference.
17. The ratio of the difference of two numbers to their sum.
18. The quotient of $(m + n)$ divided by $(p + r)$.
19. The sum of three numbers multiplied by their product.
20. $(x + y)$ multiplied by $(x - y)$ and divided by $(m + n)$.
21. The sum of y and z subtracted from x.
22. The sum of p, q, and r divided by the difference of x and y.
23. The product of p, q, and r divided by the sum of m and n.
24. The difference of the ratio of a to b and the ratio of c to d.
25. The difference of p and q subtracted from the sum of m, n, and r.
26. $(a+b)$ added to $(c+d)$ and the sum divided by $(e - f)$.

EXERCISE 3.—LETTER NOTATION

If a boy begins a game of marbles with 12 marbles, wins 3, and loses 5, how many marbles does he then possess ?

The answer is $12 + 3 - 5$, and this $= 10$.

If a boy begins a game of marbles with x marbles, wins y, and loses z, how many marbles does he then possess ?

The answer is $x + y - z$. This cannot be simplified.

Answer the following questions in the same way, first noting how you work arithmetically, then doing the same thing algebraically.

1. In a class of 30 children, 14 are boys. How many are girls ?

   In a class of a children, b are boys. How many are girls ?

2. A man went on a journey, going 65 miles by train, 4 miles by bus, and 3 miles on foot. How far did he travel ?

   A man went on a journey, going x miles by train, y miles by bus, and z miles on foot. How far did he travel ?

3. How far will a motor-car go in 3 hours at 35 miles an hour ?

   How far will a motor-car go in t hours at v miles an hour ?

4. A milkman starts on his rounds with 36 bottles of milk. He delivers 30, and 4 others get broken. With how many bottles of milk does he return ?

   A milkman starts on his rounds with m bottles of milk. He delivers n, and p others get broken. With how many bottles of milk does he return ?

5. After a girl had spent 1*s*. 6*d*. for a seat at a cinema, and 4*d*. on an ice, she had 2*d*. left. What money had she at first ?

   After a girl had spent x pence for a seat at a cinema, and y pence on an ice, she had z pence left. What money had she at first ?

6. A box contained 144 oranges. These were shared equally among 48 children. How many did each child receive ?

   A box contained p oranges. These were shared equally among q children. How many did each child receive ?

7. After giving 4 chocolates to each of 24 girls, the giver found she had 3 left. How many chocolates were there originally ?

   After giving a chocolates to each of b girls, the giver found she had c left. How many chocolates were there originally ?

8. An estate was sold for £10,000. After £500 had been deducted for lawyers' fees, the remainder was divided equally among 5 claimants. How much money did each receive ?

   An estate was sold for £x. After £y had been deducted for lawyers' fees, the remainder was divided equally among p claimants. How much money did each receive ?

9. A sack holds 14 lb. of potatoes. How many sacks will be required to hold a hundredweight of potatoes ?

   A sack holds x lb. of potatoes. How many sacks will be required to hold y lb. of potatoes ?

10. What is the average of 4, 7, 10, and 11 ? What is the average of a, b, c, and d ?

11. Reduce 10 miles to yards. Reduce n miles to yards.

12. Reduce 720 pence to pounds. Reduce q pence to pounds.

13. If the pedals of a bicycle make one complete turn while the back wheel makes 3 turns, how many turns do the pedals make when the back wheel revolves 600 times ?

    If the pedals of a bicycle make one complete turn while the back wheel makes p turns, how many turns do the pedals make when the back wheel revolves r times ?

EXERCISE 4.—FIGURES INSTEAD OF LETTERS

Look at this problem : A teacher has p pencils in a box. He gives q pencils to each pupil and has r pencils left. How many pupils has he ?

If there were figures instead of letters you could solve this quite easily. Let us therefore suppose that the question reads like this :

A teacher has 120 lead pencils in a box. He gives 3 pencils to each pupil and has 24 pencils left. How many pupils has he ?

The number of pencils given out is $120 - 24 = 96$.

$\therefore$ The number of pencils given out is $p - r$.

Since each pupil is given 3 pencils, there are $96 \div 3$ pupils.

Since each pupil is given q pencils, there are $\frac{p-r}{q}$ pupils.

Solve the following in the same way :

1. What will m books cost at x shillings each ?
2. How many pence are there in x shillings ?
3. How many pence are there in x shillings and y pence ?
4. How many pence are there in £w, x shillings, and y pence ?
5. Two places, A and B, are d miles apart. How long will it take a train to go from A to B at the rate of 40 miles per hour ? How long at r miles per hour ?
6. I keep my books in two rooms. In one room there are n shelves with 20 books on each shelf, and in the other there are p shelves with 25 books on each shelf. How many books have I ?

   If instead of 20 books on each shelf in the first room I had q books, and instead of 25 in the second room I had r, how many books should I then have ?
7. What is the area of a rectangle if its length is x inches and its breadth y inches ?
8. If I walk x miles from my home and then walk back y miles, how far am I from home ? (Assume x to be larger than y.)
9. How many yards would t inches make ?
10. From a piece of string x inches long y pieces z inches long are cut off. How long is the string that is left ?

EXERCISE 5.—BRACKETS I

Numbers in brackets should always be taken together and treated as one number. E.g. $(4 + 3)$ must always be taken as 7, so that $9 - (4 + 3)$ is not the same as $9 - 4 + 3$. The former comes to 2, but the latter comes to 8. So $a - (b + c)$ is different from $a - b + c$. If there is no danger of the numbers being taken separately, the brackets are omitted. So "the product of $(a + b)$ and $(a - b)$" may also be written "the product of $a + b$ and $a - b$." But I must not write $a + ba - b$ when I mean $(a + b)(a - b)$.

Just as $3 + 3 + 3 + 3 = 4$ threes $= 4 \times 3$
So $a + a + a + a = 4 \times a = 4a$
Just as 5 twos $= 2 + 2 + 2 + 2 + 2$
So $5x = x + x + x + x + x$
Also $3(m - n) = (m - n) + (m - n) + (m - n)$

Just as 1 apple + 2 apples + 3 apples = 6 apples
and 1 dozen + 2 dozen + 3 dozen = 6 dozen
So $a + 2a + 3a = 6a$
and $(a - b) + 2(a - b) + 3(a - b) = 6(a - b)$

Note.—We never say $1a$ or $1x$, but simply a or x.

Using letters only, and keeping brackets, write out :

1. $3a$.
2. $5n$.
3. $2(x + y)$.
4. $2(a + b - c)$.
5. $2a + 3b$.
6. $2a - 3b$.
7. $3(a - b)$.
8. $2(m + n) - (m - n)$.
9. $3ab + 4cd$.

Write out more simply, using figures as well as letters, and keeping brackets :

10. $m + m + m + m$.
11. $p + p + p + p + p$.
12. $(a + b) + (a + b)$.
13. $(c - d) + (c - d) + (c - d) + (c - d)$.
14. $(a + b - c) + (a + b - c)$.
15. $2a + 3a + 7a$.
16. $2(a + b) + 3(a + b)$.
17. $2a + b + a + 2b$.
18. $2m + 2n + 3m + n + p + p$.
19. $2(x + y) + (x + y) + 2(a - b) + (a - b)$.
20. $3(a - b + c) + 2(a - b + c) + (a - b + c)$.
21. $2(x + y - z) + (x + y + z) + 4(x + y + z)$.

EXERCISE 6.—ALGEBRAIC NOTATION

3×3 may be written 3^2, and $3 \times 3 \times 3$ may be written 3^3, and so on.

In the same way $a \times a \times a$, or aaa, may be written a^3, and abb may be written ab^2.

$\sqrt{36}$ means the square root of 36; that is, 6, because $6 \times 6 = 36$.

$\sqrt{x}$ means the square root of x; that is, the number which when multiplied by itself becomes x. If $nn = x$ then $n = \sqrt{x}$.

$\sqrt[3]{27}$ means the cube root of 27; that is to say, the number which, when three of them are multiplied together, will make 27.

Because $3 \times 3 \times 3 = 27$, $\sqrt[3]{27} = 3$. If $rrr = x$, then $\sqrt[3]{x} = r$.

In x^2 the 2 is called an *index*, the plural of which is *indices*.

In y, y^2, y^3, y^4, and y^5, 1, 2, 3, 4, and 5 are the indices of y.

Use indices to write these numbers in a simpler form:

1. 4×4. 2. $6 \times 6 \times 6$. 3. mm.
4. $nnnn$. 5. $(a + b)(a + b)$. 6. $(x - y)(x - y)(x - y)$.
7. $(a + b - c)(a + b - c)$. 8. $aaa + bbb$.
9. $aa + bb + cc - dd$. 10. $(a + b)(a + b) + (a - b)(a - b)$.
11. $\dfrac{mmm}{nn + pp}$. 12. $\dfrac{aa + bb - cc}{aab - abb + bcc}$.
13. $\dfrac{(m + n)(m + n)}{(m - n)(m - n)}$. 14. $\dfrac{x + y - z}{(x - y + z)(x - y + z)}$.

Write out in full, without using indices or the sign for multiplication:

15. 5^2. 16. 10^3. 17. x^4. 18. $(a + b)^3$. 19. $a^2 + b^2 + c^2$.
20. $a^3 - b^3$. 21. $\dfrac{(a - b)^2}{a + b}$. 22. $(m + n)^2 - (m - n)^2$.
23. $ab^2 + b^2c$. 24. $(2a + 3b)^3$.
25. $\dfrac{x^2 + y^2 + z^2}{x^2 - y^2 - z^2}$. 26. $\dfrac{a^2b + c^2d}{ab^2 - cd^2}$. 27. $(a^3b - ab^3)^3$.

Find the value of the following:

28. $\sqrt{16}$. 29. $\sqrt[3]{8}$. 30. $\sqrt{49}$. 31. $\sqrt[3]{64}$. 32. $\sqrt{x^2}$.
33. $\sqrt[3]{a^3}$. 34. $\sqrt{(a + b)^2}$. 35. $\sqrt[3]{(x + y)^3}$.

EXERCISE 7.—THE USE OF LETTERS

1. How many pence are there in 5 shillings? How many in x shillings?
2. If a book costs 10 shillings, how many shillings must I pay for 6 of them? How many for n of them?
3. If I walk at a steady rate of 3 miles an hour, how many miles shall I walk in 4 hours? How many in t hours?
4. The area of a rectangle is 15 sq. in. Two of the sides are 5 in. long; how many inches long are the others? How many if two of the sides are n in. long?
5. When the diameter of a circle is 2 in. long, how many inches long is the circumference? How many when there are x in. in the diameter? (Write π instead of 3·1416.)
6. What in algebraic form is the rule for finding the area (A) of a rectangle when the length (l) and the breadth (b) are given?
7. Write in algebraic symbols the rule for finding the breadth of a rectangle when the area and the length are known. (Begin $b = \ldots$)
8. Write in letters the rule for finding the cost in shillings (c) of n things when the price in shillings (p) of one of them is given.
9. A man is 30 years older than his son. Write the formula for finding the father's age (f years) when the son's age (s years) is known. (Begin $f = \ldots$)
10. A man was 35 when his son was born. Write in algebraic form the rule for finding the son's age when the father's age is known. (Begin $s = \ldots$)
11. Write the rule for changing shillings into pounds. (Let x = the number of pounds and n = the number of shillings. Begin $x = \ldots$)
12. At 40 minutes past 10 o'clock how many minutes must pass before it is 11 o'clock? How many at x minutes past 10?
13. Express n half-crowns in pounds sterling.
14. From a piece of string p yds. q ft. long, a piece r ft. long is cut off. How many inches of string remain?

EASY TEST 1

1. Express algebraically the product of two numbers.
2. Write the following in the simplest way : (i) $3 \times a \times b$, (ii) xx, (iii) m divided by n.
3. Express as a fraction the sum of two numbers divided by their difference.
4. What is the difference of $m + n$ and $p + r$?
5. To x add the sum of y and z.
6. How many inches are there in x ft. ?
7. How many feet are there in x in. ?
8. If a boy walks at the rate of 3 miles an hour, how many miles will he go in t hours ? How many miles will he go in t hours at v miles an hour ?
9. What will x pounds of butter cost at y shillings a pound ?
10. A, B, and C are three posts standing in a line. If the distance from A to B is n yards, and the distance from B to C is twice as great, what is the distance from A to C ?
11. There are c children present in a school. If a of them are at work in the classrooms, b are singing in the hall, and the rest are drilling in the playground, how many are in the playground ?
12. If there are p pages in a book, on an average w words on each page and l letters in each word, how many letters are there in the book ?
13. How ought the following to be written : (i) $p \times 25$, (ii) $4 \times a + 7$, (iii) $n \times 2 - 5$?
14. What is the value of : (i) $7x$ when $x = 8$, (ii) $8a$ when $a = \frac{1}{2}$, (iii) $5x + 3$ when $x = 2$?
15. Write out in full without using indices : (i) p^3, (ii) $a^2b - ab^2$, (iii) $(x + y)^2 - (x - y)^2$.
16. How many pence are there in x pounds y shillings ?
17. Show how one " s " differs from the other in the statement : Silk costs ss. a yard.
18. Show how one " lb." differs from the other in these two statements :

    (i) 16 oz. = 1 lb., (ii) $A = lb$.

1. Write in the simplest way: (i) $5 \times a \times a \times b$, (ii) $(a - b)(a - b)(a - b)$, (iii) $x - y$ divided by $x + y$.
2. Write out in full without using indices or " $\times$ ": (i) $x^4 + 3x^3y - 5x^2y^2 + 6xy^3 - 7y^4$, (ii) $(x^2 - y^2)^2$.
3. Write out more simply, using figures as well as letters: (i) $p + p + 2p$, (ii) $2(m + n) + (m + n)$, (iii) $5(x - y + z) - 3(x - y + z)$.
4. Without using the sign " $\times$ " express the product of $a + b$ and $c + d$.
5. Without using " $\div$ " express the sum of a, b, c, and d divided by their product.
6. From the sum of x and y take the difference of p and q.
7. How many seconds are there in (i) p minutes, (ii) $\frac{p}{2}$ minutes?
8. What is the total cost of x loaves at y pence each, m pints of milk at n pence a pint, and p pounds of cheese at r pence a pound?
9. A class of p boys is drilled in q rows. How many boys stand in a row?
10. In a book of p pages, with an average of q words to the page, r pages have been torn out. How many words remain?
11. If I travel d hours in a train which goes at the rate of v miles an hour, and then walk n hours at p miles an hour, how far do I go altogether?
12. What is the value of: (i) $5x + 4$ when $x = 9$, (ii) $\frac{4x}{5}$ when $x = 15$, (iii) $\sqrt{13x}$ when $x = 13$?
13. Express entirely in words: (i) $a + b + c + d$, (ii) $(p - r)$, (iii) xyz, (iv) $\frac{x}{y}$.
14. Put $a + b - c$ entirely into words.
15. A book has p pages. After q pages are read, what fraction of the book remains unread?

EXERCISE 8.—EXPRESSIONS

$ax^2 + bx - c$ is an *expression* containing three *terms*.

The terms of an expression are connected by plus or minus signs.

The value of an expression depends on the value of the letters used.

For example, $a + b = 6$ when $a = 2$ and $b = 4$. But when $a = 10$ and $b = 15$, $a + b = 25$.

The numbers to be multiplied together to form a term are called its *factors*. Thus 5, a, and b are the factors of the term $5ab$.

1. What is the value of $m - n$ if $m = 12$ and $n = 4$?
2. What is the value of $m - n$ if $m = 20$ and $n = 7$?
3. What is the value of $4x$ if $x = 3$?
4. What is the value of $3x + 2$ if $x = 5$?
5. If $a = 3$, evaluate (that is, find the value of) a^3.
6. If $a = 4$, evaluate $3a^2$.
7. Substituting 5 for x and 4 for y, find the value of $2x^2 + y$.
8. What is the value of $(a+b)(a-b)$ when $a=6$ and $b=4$?
9. Evaluate $7(x + y)$ when $x = 3$ and $y = 2$.
10. Evaluate $\dfrac{m+n}{m-n}$ when $m = 5$ and $n = 3$.
11. Evaluate $\dfrac{(m+n)^2}{(m-n)^2}$ when $m = 4$ and $n = 1$.
12. What does $5a(2b \quad c)$ amount to when $a = 2$, $b = 3$, and $c = 4$? What is its value when $a = 5$, $b = 2$, and $c = 3$?
13. When $x = 0$, what is the value of: (i) $5 + x$, (ii) $a + x$, (iii) $4x$, (iv) x^3 ?
14. Find the value of $\sqrt{a} + \sqrt{b} + \sqrt[3]{c}$ when $a = 4$, $b = 25$, $c = 27$.
15. When $x = 4$, $y = 2$, and $z = 0$, find the value of $x^3 + 3x^2y - 6xy^2 + z^3$.
16. When $x = 1$, and $y = 0$, find the value of the following:

$$x^3 + \frac{y}{x} - x^2 + xy - \frac{3y}{x^4}$$

EXERCISE 9.—EXPERIMENTS WITH BRACKETS

Remember that when numbers are enclosed in brackets they must be taken as single numbers. Thus brackets show which operations should take place first. Thus $8 - (5 - 2)$ means that the 2 must be taken from the 5 first of all. That is, the whole expression $= 5$. It is not $8 - 5 - 2$, for that $= 1$. You must not take 5 from 8, but you must take $(5 - 2)$ from 8.

Suppose I wish to find out whether $a(b + c) = ab + ac$, I can easily do so by substituting any number I like for a, any number I like for b, and any number I like for c. Let $a = 2$, $b = 3$, and $c = 4$, though any other numbers would do just as well. Then the expression becomes :

$$2(3 + 4) = 2 \times 3 + 2 \times 4$$

And this is true, for each expression (the one on the left of $=$, and the one on the right) comes to 14.

Some of the following expressions are true and some are false. Test them by putting numbers instead of the letters, and then state whether each example is true or false :

1. $a + b = b + a$.
2. $a - b = b - a$.
3. $(a + b)^2 = a^2 + b^2$.
4. $(a + b)^2 = a^2 + 2ab + b^2$.
5. $(a - b)^2 = a^2 - b^2$.
6. $(a - b)^2 = a^2 - 2ab + b^2$.
7. $a^3 + b = a + b^3$.
8. $a(a + b) = a^2 + ab$.
9. $5(x + 2y) = 5x + 10y$.
10. $5ab = 5a + 5b$.
11. $5ab = 5a \times 5b$.
12. $(a + b)(a - b) = a^2 - b^2$.
13. $abc = bca$.
14. $(x + y)(x - y) = (x - y)(x + y)$.
15. $a + b + c = c + b + a$.
16. $a + b - c = a - c + b$.
17. $a - (b - c) = a - b + c$.
18. $x - (y + z) = x - y - z$.
19. $a^2 - 2b(c + d) = a^2 - 2bc - d$.
20. $a^2 - 2b(c + d) = a^2 - 2bc - 2bd$.

EXERCISE 10.—LIKE AND UNLIKE TERMS

(i) 5 in. + 2 in. + 6 in. is an expression of *like* terms.

(ii) 5 yards + 2 ft. + 6 in. is an expression of *unlike* terms.

(iii) $5a + 2a + 6a$ is an expression of *like* terms.

(iv) $5a + 2b + 6c$ is an expression of *unlike* terms.

(v) $5a^2 + 2a + 6$ is also an expression of *unlike* terms.

Like terms can be added or subtracted and expressed as one term.

For instance (i) = 13 in. and (iii) = $13a$. The other three expressions cannot be simplified. They are already the simplest forms of the expressions.

Consider this expression :

$$5a + 3b - 2a + c + 2b + 3c - a - 4c$$

Bring like terms together, thus :

$$5a - 2a - a + 3b + 2b + c + 3c - 4c$$

This when simplified becomes $2a + 5b$.

Some of the following expressions can be simplified ; others cannot. Simplify those that can, and copy out the others.

1. $6a + 2a$.
2. $6a - 2a$.
3. $5ab + 2ab - 3ab$.
4. $5ab + 2bc - 3ac$.
5. $7x + 2y + 3x$.
6. $3x^2 - 2x^2 + x$.
7. $3mn - mn$.
8. $p \times 5 + 2p$.
9. $a^2b + 2ab^2 - a^2b$.
10. $2x^2y^2 - xy - x^2y^2$.
11. $m + m + m + m$.
12. $2x + 2x + 2x$ . . . ten terms.
13. $4a^3 + 8a^3 - 3a^3$.
14. $3x^2 + 4x^2 - 6x$.
15. $14x + 2y + z + 2x - 2y + 2z + 4y - z$.
16. $5x^2 + 3x + 2 - 3x^2 - x - 1$.
17. $3x^2 + 4 + 4x - 3 + 2x - x^2$.
18. $5a^3 + 3a^2b + 3ab^2 + b^3 - 3a^2b - 2ab^2$.
19. $6m^2 + 3mn + n^2 - 2m^2 - n^2 - 3mn$.
20. $3p^2q + 5q^3 + 2pq^2 + 3p^3 + 2p^2q - 2pq^2$.
21. $4r^4 + 3r^2 + 7r^3 + 8r - 3r^2 - 4r^3 - 4r^4$.
22. $5a^2 + 4b^2 + 3c^2 + 2b^2 - 4a^2 - 6b^2 - 3c^2$.

EXERCISE 11.—BRACKETS MAKE BUNDLES

We have seen that when an expression is put between brackets it must be regarded as a term, or even as a factor of a term. Thus $(a - b)$ may be treated as though it were one letter, such as m, or p, or x; and $(a - b)(c + d)$ as though it were mn or xy.

Just as $5a \quad + 8a \quad - 3a \quad = 10a$,

So $5(x + y) + 8(x + y) - 3(x + y) = 10(x + y)$.

Without untying the bundles (that is, without removing the brackets), simplify the following by adding or subtracting like terms:

1. $2(a + b) + 3(a + b)$.
2. $3(x + y) + 5(x + y) + 6(x - y) - 5(x - y)$.
3. $4(p + q) + (p + q) - 2(q + r)$.
4. $15(m + n) - 7(m + n) + (x + y)$.
5. $5(a + b - c) + 4(a + b - c) - 3(a + b - c)$.
6. $3\left(\frac{m - n}{p}\right) + 6\left(\frac{m - n}{p}\right) - 4\left(\frac{m - n}{p}\right) + \frac{m - n}{p}$.
7. $3(a + b) + 5(a^2 + b^2) - (a + b) - 4(a^2 + b^2)$.
8. $3(a + b) + 5(a + b)^2 - (a + b) - 4(a + b)^2$.
9. $2(x - y)^3 + 3(x - y)^2 - (x - y)^3 + (x - y)^2$.
10. $6(m + n - p)^2 + 3(m + n - p) - 3(m + n - p)^2$.
11. $4\left(\frac{x}{y}\right)^2 + 3\left(\frac{x}{y}\right) - 3\left(\frac{x}{y}\right)^2 - 2\left(\frac{x}{y}\right)$.
12. $6\sqrt{x} + 2\sqrt{y} - 4\sqrt{x} + 3\sqrt{y} - \sqrt{x} - 4\sqrt{y}$.

Look at 6 above. It may be written thus:

$$\frac{3(m - n)}{p} + \frac{6(m - n)}{p} - \frac{4(m - n)}{p} + \frac{m - n}{p}.$$

Now simplify these two:

13. $\frac{5(a + b)}{a - b} - \frac{3(a + b)}{a - b} + \frac{4(a - b)}{a + b} - \frac{3(a - b)}{a + b}$.
14. $\frac{x}{x + y - z} + \frac{7x}{x + y - z} - \frac{2x}{x + y - z}$.

In this test keep the brackets as they are.

1. How many terms has the expression :
$$a^3 + 3a^2b + 3ab^2 + b^3 \ ?$$
2. What is the value of x^3 if $x = 4$?
3. Evaluate $\dfrac{a-b}{a+b}$ where $a = 6$ and $b = 4$.
4. When $x = 0$, what is the value of $3x^2 + 2x + 5$?
5. Is this statement true ?
$$x^2 + y^2 = (x + y)(x - y).$$
6. Simplify : $5ab + 3ab - 2ab + b^2$.
7. Simplify : $a + 2a + b + b^2 + a^2 + b$.
8. Simplify : $5(x - y) + 2(x - y)$.
9. Simplify : $2(m - n) + 3(m - n) - (m + n)$.
10. Simplify : $2\left(\dfrac{a+b}{c}\right) + 3\left(\dfrac{a+b}{c}\right) - 4\left(\dfrac{a+b}{c}\right)$.
11. Simplify : $\dfrac{4}{x+y} + \dfrac{3}{x+y} + \dfrac{7}{x+y}$.
12. What are the factors of the term $6xy$?
13. What are the factors of the term $a(x + y)(x - y)$?
14. By how much does a exceed 12 ?
15. By what must m be multiplied to make 20 ?
16. If a boy eats x apples in y days, what is the average number per day ?
17. What is the quotient when $m + n$ is divided by $p + r$?
18. How would you represent the sum of three numbers divided by their product ?
19. A French kilogramme is equal to 2·2 lb. How many kilogrammes are equal to 11 lb. ?

    A French gramme is equal to a oz. How many grammes are equal to b oz. ?
20. Ten posts stand in a line 8 yards apart. How many yards is the last distant from the first ?

    x posts stand in a line y yards apart. How many yards is the last distant from the first ?
21. How many cwt. are there in x tons y cwt. ?

HARDER TEST 2

In this test keep the brackets undisturbed.

1. How many factors are there in the term $5ab(x + y)$? State what they are.
2. What is the value of $(x + y)^2$ when $x = 5$ and $y = 6$?
3. Evaluate the following when $a = 5$ and $b = 3$:
$$\frac{a+b}{a-b} + \frac{a-b}{a+b}.$$
4. Find the value of $\dfrac{(p+r)^2}{(p-r)^2} - \dfrac{p}{r}$ when $p = 3$ and $r = 1$.
5. What two numbers are stated to be equal in the following if $x = 4$?
$$x^2 + 2x + 1 = (x + 1)^2$$
6. Is this statement true?
$$a - b - c + 4 = 4 - (c + b) + a.$$
If so, what two numbers are stated to be equal when $a = 5$, $b = 2$, and $c = 1$?
7. Simplify $5(m + n) + 2(m - n) - 3(m + n) - (m - n)$.
8. Simplify $4\left(\dfrac{a-b}{a+b}\right) + 3\left(\dfrac{a-b}{a+b}\right) - 7\left(\dfrac{a-b}{a+b}\right)$.
9. If 3 is a factor of x, what is the other factor? If y is a factor of x, what is the other factor?
10. If 5 times x is y, what is the value of x? If p times x is y, what is the value of x?
11. What is the quotient when the sum of p, q, and r is divided by the difference of x and y?
12. Add the ratio of a to b to the ratio of c to d.
13. If I have in my pocket x half-crowns and y florins, how many shillings have I?
14. If a boy was n years old 3 years ago, how old will he be in p years' time?
15. A child covers c ft. at each step. How many steps will he take to walk round the edge of a lawn d ft. square?
16. If a sum of a pounds b shillings is divided equally among c children, how many shillings does each receive?

If $x = 10$, the expression $3x^2 + 6x + 5$

$= 3 \times 10^2 + 6 \times 10 + 5 = 300 + 60 + 5 = 365.$

If $x = 10$, the expression $2x^3 + 8x + 4$

$= 2{,}000 + 80 + 4 = 2{,}084.$

Substitute 10 for x in the following expressions and find their value :

1. $x + 4$.
2. x^2.
3. $3x + 5$.
4. $x^2 + 4x + 6$.
5. $7x^2 + 8x + 2$.
6. $5x^2 + 3$.
7. $6x^2 + 5x$.
8. $8x^3 + 4x^2 + x + 9$.
9. $5x^3 + 3x^2 + 4$.
10. $x^4 + 3x^3 + 7x^2 + 4x + 2$.

Write the following numbers in the form of algebraic expressions in which $x = 10$:

11. 16.
12. 50.
13. 836.
14. 2,914.
15. 5,870.
16. 83,725.
17. 50,638.
18. 37,201.

x^4 is called the fourth *power* of x, x^3 the third *power* of x, etc. Arrange the following terms in descending powers of x, e.g. $x + x^3 + 3 + 2x^2$ should be $x^3 + 2x^2 + x + 3$.

19. $5 + 3x^2 + 7x^3 + 2x$.
20. $4x^2 + 3 + x^5 + 7x$.
21. $2x^3 + 8 + 4x^2 + x^5 + 9x^4$.
22. $3x^6 + 8 + 5x^4 + 9x + 4x^5 + 6x^7 + 2x^3$.

23–26. Write out the values of the last four expressions when $x = 10$.

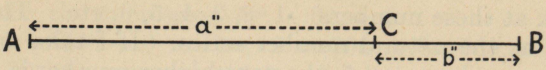
27. If $x = 5$ find the value of :
    (i) $3x^2 + 2x + 4$.
    (ii) $4x^3 + 3x^2 + x + 2$.
    (iii) $x^4 + 2x^3 + 3x^2 + 4x + 3$.
28. If $x = 8$ find the value of :
    (i) $3x^2 + 7x + 5$.
    (ii) $x^3 + 4x^2 + 6x + 2$.
    (iii) $3x^4 + 2x^3 + 7x^2 + 5x + 6$.
29. If $x = 12$ find the value of :
    (i) $2x^2 + 5x + 4$.
    (ii) $3x^3 + 9x^2 + 10x + 3$.
    (iii) $7x^3 + 11x^2 + 6x + 7$.
    (iv) $8x^3 + 10x^2 + 2x + 9$.

EXERCISE 13.—THE NUMBER SERIES

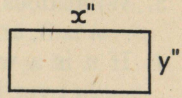
Look at these numbers : 1, 2, 3, 4, 5, 6, etc. They are known as the natural number series. If I take any one of these numbers, say 5, the next higher number is $5 + 1$, or 6, and the next lower number is $5 - 1$, or 4.

1. What is the next higher number to n ?
2. What is the next lower number to n ?
3. What does $2n$ become when n is 1, when n is 2, when n is 3, and when n is 4 ?
4. If n is a whole number (that is, not a fraction), can $2n$ ever be an odd number ?
5. What is the value of $2n + 1$ when n is 1, when n is 2, when n is 3, and when n is 4 ?
6. What is the value of $2n - 1$ when n is 5, when n is 6, when n is 7, and when n is 8.
7. Assuming that n is a whole number, say whether $2n + 1$ is always odd or always even.
8. Is $2n - 1$ always odd ?
9. By what algebraic term can we express an even number ?
10. By what algebraic expression can we indicate an odd number ?
11. Add together the number n, the next higher number, and the next lower number.
12. What is the value of any five numbers in the natural number series, of which n is the middle number ?
13. What is the sum of the first two odd numbers (1 and 3) ?
14. What is the sum of the first three odd numbers ?
15. What is the sum of the first four odd numbers ?
16. What is the sum of the first n odd numbers ?
17. What is the 2nd number in the series 3, 6, 9, 12, etc. ?
18. What is the 5th number of the series given in question 17 ?
19. What is the 20th number of the series given in question 17 ?
20. What is the nth number of the series given in question 17 ?

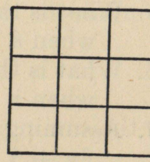
EXERCISE 14.—PROBLEMS



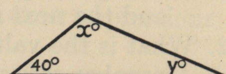
1. Here is a line 12 in. long. Without using a, give the length of AC in inches.
2. Without using b, give the length of CB in inches.
3. How long is $a + b$?
4. Here is a rectangle. What is its area in square inches ?



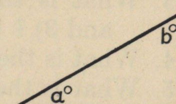
5. How many inches round the rectangle ?
6. Here is a network of nine squares. The side of each mesh (each little square) is x in. What length of line is necessary to form one mesh ?



7. What is the perimeter (the distance round) of the large square ?
8. What is the area of one mesh ?
9. What is the area of the whole figure ?
10. What length of line is there in the whole figure ?
11. In this triangle the sizes of the angles are marked in degrees. The sum of the three angles of every triangle is 180°. How many degrees in $x + y$?



12. How many degrees is x in terms of y ?
13. How many degrees is y in terms of x ?
14. This is a right-angled triangle. How many degrees are there in $a + b$?



15. Express the number of degrees in a in terms of b.
16. Express the number of degrees in b in terms of a.
17. What is the perimeter (distance round) of a rectangle which is x in. broad and is twice as long as it is broad ? What is its area ?
18. What is the perimeter of a rectangle which is x in. long and is twice as long as it is broad ? What is its area ?

EXERCISE 15.—ONE AND NOUGHT

Note these peculiarities of the number 1:

$1^2 = 1$; $1^3 = 1$; $1^4 = 1$; $1^5 = 1$, $1^n = 1$. To whatever power 1 is raised, it still remains 1.

$\sqrt{1} = 1$; $\sqrt[3]{1} = 1$; $\sqrt[4]{1} = 1$; $\sqrt[5]{1} = 1$; $\sqrt[n]{1} = 1$. All roots of 1 are 1.

In a later exercise it will be proved that $1^0 = 1$; $2^0 = 1$; $x^0 = 1$. Whatever number has 0 as an index $= 1$.

The number 1 is not always expressed, e.g. x means $1x$; $a + b$ means $1(a + b)$ or $1a + 1b$; $1^1 = 1$; $x^1 = x$. An index of 1 does not change the value of a number.

When we cancel, as in $\dfrac{\overset{a}{\cancel{a^2}}b}{\cancel{a}}$, a blank means not 0 but 1.

In the example the result is not $\dfrac{ab}{0}$, but $\dfrac{ab}{1}$ or ab.

$4 = \dfrac{4}{1}$; $25 = \dfrac{25}{1}$; $x = \dfrac{x}{1}$; $a + b = \dfrac{a + b}{1}$. Where it is necessary to treat a whole number as a fraction we give it 1 as a denominator.

$1 \times 0 = 0$; $0 \times 7 = 0$; $0x = 0$; $0abc = 0$; $0(x + y)(p + r) = 0$. If 0 is the factor of a term the whole term $= 0$.

The expression $x^2 + 3 + 5x^4 - 4x - 2x^3$ is disorderly. It should be written in *descending* powers of x, thus: $5x^4 - 2x^3 + x^2 - 4x + 3$. If you wish to show that x is virtually present in every term you can write the expression thus: $5x^4 - 2x^3 + x^2 - 4x^1 + 3x^0$.

The expression in *ascending* powers of x becomes:

$$3 - 4x + x^2 - 2x^3 + 5x^4$$

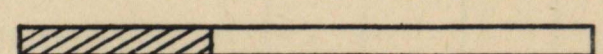
Without removing brackets, find the value of:

1. 1^{12}. 2. 1^p. 3. $\sqrt[6]{1}$. 4. $\sqrt[r]{1}$. 5. $3 \times 6 \times 0$.
6. $3 \times 6 \times 1$. 7. $0xy$. 8. $1xyz$. 9. $0(x - y)^2$. 10. $1(x + y)^2$.

Arrange the following in descending powers of x:

11. $5x - 4x^2 - 3 + 6x^3$.
12. $2x^4 + x^6 - x^3 + 7 - 3x - 3x^5 + 5x^2$.
13. Re-write your answer to question 11 to show that x is virtually present in every term.

EASY TEST 3

1. If $x = 10$, what is the value of: (i) $3x^2 + 6x + 5$, (ii) $8x^2 + 2$?
2. Write the following numbers in the form of algebraic expressions in which $x = 10$: (i) 24, (ii) 204, (iii) 240, (iv) 5,036.
3. What number in the natural number series is next but one above n? What number is next but one below n?
4. How would you express an odd number ?
5. Name the first four odd numbers.
6. What is the 10th number of the series: 5, 10, 15, 20, etc. ?
7. If this ribbon is x in. long  and the shaded part is y in. long, how long is the unshaded part ?
8. If the side of a square is p in., what is its area ?
9. What is the perimeter of a square if its side is p in. ?
10. What is the perimeter of a rectangle if its length is m in. and its breadth n in. ?
11. If one angle of a triangle is a degrees, what is the sum of the other two angles ?
12. Arrange this expression in descending powers of y:
$$y - 2y^3 - 6 + 3y^4 - 7y^2.$$
13. What is the quotient when the product of p, q, and r is divided by the sum of m and n ?
14. If $a = 2$ and $b = 3$, what is the value of this expression: $a^3 + a^2b + ab^2 + b^3$?
15. If $x = 4$ and $y = 3$, what is the value of:
$$\frac{x + y}{x - y} + \frac{x - y}{x + y}?$$
16. From m subtract the difference of p and q.
17. A school has 7 classrooms. In each classroom there are x boys and y girls. How many children are there in the school ?
18. Write down the three consecutive numbers of which p is the least.
19. What is the value of $a^3 + a^2b + ab^2 + b^3$ when $b = 0$?

1. If $x = 10$, what is the value of $x^5 + 7$?
2. Write as an algebraic expression in which $x = 10$: (i) 5,893, (ii) 20,060.
3. Write down, starting with the lowest, four consecutive numbers of which the greatest is p.
4. What is the sum of the first 6 odd numbers ?
5. What is the nth number of the series 2, 4, 6, 8, etc. ?
6. If the area of a rectangle is x sq. in. and its length is y in., how broad is it ?
7. If a rectangular wall a ft. long and b ft. high has in it 2 windows each c ft. by d ft., what area of wall is left to be distempered ?
8. If a triangle has a right angle, and one of the other angles is n degrees, how many degrees is the third angle ?
9. Arrange the following terms in proper order, and then find the value of the expression if $a = 3$ and $b = 2$:

$$2a^2b^2 - a^3b + 5b^4 + 3a^4 - 4ab^3.$$

10. Evaluate this expression when $x = 5$ and $y = 2$:

$$4x^3 - (x^2y - xy^2 + y^3).$$

11. If $a = 7$ and $b = 3$, what is the value of the following?

$$\frac{a+b}{a-b} + \frac{a-b}{a+b} + \frac{1}{10}$$

12. What is the interest on £x in 1 year at r per cent. per annum ?
13. What is the interest on £x in t years at r per cent. per annum ?
14. What is the sum of 7 consecutive numbers of which the middle one is m ?
15. Evaluate the following when $a = 2$, $b = 1$, $c = 0$:

$$6a^2 - 2ab + 3ac - 5bc + abc.$$

16. Evaluate the following when $x = 3$, $y = 1$, $z = 0$:

$$\frac{x}{y} + \frac{z}{x} + \frac{x-y}{x+z-1} - \frac{z}{x+2} + y^2.$$

EXERCISE 16.—ADDITION

Compare these two addition sums :

$$\begin{array}{rrrr} 2, & 4 & 1 & 5 \\ 3, & 0 & 4 & 1 \\ 8, & 5 & 2 & 0 \\ \hline 13, & 9 & 7 & 6 \\ \hline \end{array} \qquad \begin{array}{l} 2x^3 + 4x^2 + x + 5 \\ 3x^3 + 4x + 1 \\ 8x^3 + 5x^2 + 2x \\ \hline 13x^3 + 9x^2 + 7x + 6 \\ \hline \end{array}$$

Now compare these two :

$$\begin{array}{rrrr} 6, & 9 & 3 & 7 \\ & 2 & 6 & 5 \\ 8, & 3 & 7 & 6 \\ & & 5 & 9 \\ \hline 15, & 6 & 3 & 7 \\ \hline \end{array} \qquad \begin{array}{rrrr} 6x^3 + & 9x^2 + & 3x + & 7 \\ & 2x^2 + & 6x + & 5 \\ 8x^3 + & 3x^2 + & 7x + & 6 \\ & & 5x + & 9 \\ \hline 14x^3 + & 14x^2 + & 21x + & 27 \\ \hline \end{array}$$

In our number system $x = 10$. Putting 10 for x in the last sum it becomes $14{,}000 + 1{,}400 + 210 + 27 = 15{,}637$.

In arithmetic we carry from one column to another, because we know that x is 10. In algebra we cannot carry, because we do not know the value of x. It may be 5, or 12, or 20, or indeed any number whatever. Just as in arithmetic we put all the tens under one another, and all the hundreds under one another, for convenience of adding, so in algebra we put all the *like* terms under one another for convenience of adding.

Add together :

1. $2x + 3$ and $4x + 5$.
2. $5x^2 + 6x + 3$ and $8x + 7$.
3. $5x + 7$ and $8x$.
4. $4x^2 + 6$ and $9x + 1$.
5. $3x + 7$, $2x + 4$, $9x + 6$, $5x + 3$, and $7x + 1$.
6. $5x^2 + 7x + 3$, $2x^2 + 9x + 8$, $6x^2 + 4x + 16$, and $x + 10$.
7. $x^3 + 3x^2 + 9x + 2$, $8x^3 + 7x^2 + 10x + 5$, and $3x^3 + 2x$.
8. $6x^4 + 7x^3 + 8x^2 + 6x + 3$ and $12x^4 + x^3 + 5x^2 + x + 13$.

Simplify the following :

9. $(x + 3) + (2x + 9) + (3x + 5) + (x + 2) + (6x + 6)$.
10. $(x^2 + x + 1) + (5x^2 + 3x + 7) + (6x^2 + 4) + (4x + 3) + (2x^2 + x)$.
11. $(3x^3 + 3) + (2x^2 + 2) + (x + 1) + (5x^3 + 4x^2 + 3x + 2) + (7x^2 + 8x)$.
12. $(4x^5 + 2x^2) + (8x^3 + 7x^2) + (3x^4 + 7x^3) + (5x^2 + 3) + (3x^3 + 5x)$.

EXERCISE 17.—MULTIPLICATION I

Although I cannot simplify $x^2 + x^3$, I can simplify $x^2 \times x^3$. For $x^2 \times x^3 = x \times x \times x \times x \times x = x^5$. Note that the index of the product is the sum of the indices of the factors. Note also that the index of x is 1, for $x = x^1$.

Multiply :

1. a by a^2.
2. m^2 by m.
3. p^2 by p^2.
4. x by x^4.
5. ab by a.
6. a^2b by a.
7. ab^2 by a.
8. ab by ab.
9. a^2b^2 by a^2b^2.
10. xy by y^2.
11. x^3 by x^2y.
12. x^4y^3 by x^3y^4.
13. xyz by xy.
14. x^2yz by xyz^2.
15. $x^2y^2z^2$ by x^2y.

Look at this expression: $x^3 + 3x^2 + 4x + 2$. The *coefficient* of x^3 is 1, of x^2 is 3, and of x is 4. If $3x^2$ is multiplied by 2, the result is $6x^2$; if multiplied by $2x$, it is $6x^3$.

Multiply :

16. $2a$ by 4.
17. $5a^2$ by 3.
18. $3a$ by a.
19. $5x^3$ by $3x$.
20. x^5 by $4x^3$.
21. $2x^2y$ by $3xy^2$.

Compare these two multiplication sums :

$$\begin{array}{rrr} 4 & 2 & 3 \\ & & 2 \\ \hline 8 & 4 & 6 \\ \hline \end{array} \qquad\qquad \begin{array}{r} 4x^2 + 2x + 3 \\ 2 \\ \hline 8x^2 + 4x + 6 \\ \hline \end{array}$$

Compare these two sums :

$$\begin{array}{rrr} 5 & 8 & 7 \\ & & 6 \\ \hline 3{,}5 & 2 & 2 \\ \hline \end{array} \qquad\qquad \begin{array}{r} 5x^2 + \ \ 8x + 7 \\ 6 \\ \hline 30x^2 + 48x + 42 \\ \hline \end{array}$$

If x is 10, the answer to the last sum is $3{,}000 + 480 + 42$, which equals 3,522.

Multiply :

22. $5x + 3$ by 3.
23. $6x^2 + 3x + 1$ by 7.
24. $2x^2 + 3x + 4$ by $2x$.
25. $4x^3 + 2x + 7$ by $4x$.
26. $5x^2 + 4x + 3$ by $5x^2$.
27. $x^4 + 2x^2 + 5$ by $6x^3$.
28. $3x^3 + 5x^2 + 7x$ by $4x^3$.
29. $3x^6 + 6x^4 + 9x^2$ by $7x^2$.

EXERCISE 18.—MULTIPLICATION II

Compare these two multiplication sums :

$$\begin{array}{rrrr} & 5 & 4 & 6 \\ & & 3 & 2 \\ \hline 16, & 3 & 8 & \\ 1, & 0 & 9 & 2 \\ \hline 17, & 4 & 7 & 2 \end{array} \qquad \begin{array}{rrrr} & 5x^2 + & 4x + & 6 \\ & & 3x + & 2 \\ \hline 15x^3 + & 12x^2 + & 18x & \\ & 10x^2 + & 8x + & 12 \\ \hline 15x^3 + & 22x^2 + & 26x + & 12 \end{array}$$

If $x = 10$, then the answer to the second sum $= 17,472$. Since in algebra we cannot carry numbers from one power of x to another, there is no advantage in beginning to work the sum from the smaller end ; that is, from the right. In algebra we work from left to right, thus :

$$\begin{array}{llll} 5x^2 + & 4x & + 6 & \\ 3x & + 2 & & \\ \hline 15x^3 + & 12x^2 + & 18x & \\ & 10x^2 + & 8x + & 12 \\ \hline 15x^3 + & 22x^2 + & 26x + & 12 \end{array}$$

Multiply :

1. $3x + 5$ by $2x + 1$. 2. $6x + 7$ by $9x + 8$.
3. $5x^2 + 6x + 3$ by 5. 4. $3x^2 + 7x + 1$ by $6x + 4$.
5. $4x^2 + 2$ by $6x + 2$. 6. $7x^3 + x^2 + 2x + 5$ by $3x + 4$.
7. $2x^2 + 3x + 5$ by $3x^2 + x + 2$.
8. $5x^3 + 2x^2 + x + 6$ by $x^2 + 3x + 4$.

Compare these multiplication sums :

$$\begin{array}{rrrr} & 1 & 0 & 2 \\ & & 3 & 4 \\ \hline 3 & 0 & 6 & \\ & 4 & 0 & 8 \\ \hline 3 & 4 & 6 & 8 \end{array} \qquad \begin{array}{llll} x^2 + 2 & & & \\ 3x + 4 & & & \\ \hline 3x^3 & & + 6x & \\ & 4x^2 & & + 8 \\ \hline 3x^3 + & 4x^2 + & 6x + & 8 \end{array}$$

The above multiplicand might be written $x^2 + 0x + 2$.

Multiply :

9. $2x^2 + 3$ by $x + 1$. 10. $5x^3 + x$ by $x + 2$.
11. $3x^3 + 2$ by $2x + 3$. 12. $4x^2 + 2$ by $5x + 7$.
13. $5x^4 + 2x^3 + 3x + 1$ by $x + 4$.
14. $3x^4 + 4x^2 + 5x + 2$ by $2x + 5$.
15. $2x^4 + 3x^2 + x + 4$ by $x^2 + 1$.

EXERCISE 19.—MIXED EXPRESSIONS

Compare these two expressions :

3 yd. 2 ft. 8 in. and $3a + 2b + 8$.

Let $a = 36$ and $b = 12$, then the latter expression can stand for the number of inches in 3 yd. 2 ft. 8 in.

Compare these two expressions :

£4 12*s*. 6*d*. and $4a + 12b + 6$.

Note that the letters £, *s*., and *d*. are not algebraic symbols at all ; for algebraic symbols always stand for *numbers*, but £, *s*., and *d*. stand for words—the words pounds (*libræ*), shillings, and pence (*denarii*).

Let $a = 240$ and $b = 12$, then the expression—

$$4a + 12b + 6$$

indicates the number of pence in £4 12*s*. 6*d*.

What weights, measures, or sums of money do the following expressions represent ? The answer should take this form : The number of . . . in . . . .

1. $4a + 6$, when $a = 12$. (There are two answers.)
2. $2a + 30$, when $a = 60$. (Two answers.)
3. $4x + 1{,}425$, when $x = 1{,}760$.
4. $5m + 15n + 8$, when $m = 240$ and $n = 12$.
5. $2a + 3b + 1$, when $a = 8$ and $b = 2$.
6. $7p + 3q + 5$, when p $= 28$ and $q = 7$.
7. $5a + 8b + 60$, when $a = 1{,}296$ and $b = 144$.
8. $6x + 8y + 12$, when $x = 224$ and $y = 16$.

Evaluate the following :

9. $3a + 146$, when $a = 1{,}760$.
10. $6a + 7b + 1$, when $a = 320$ and $b = 40$.
11. $5x + 2y + 10$, when $x = 36$ and $y = 12$.
12. $5x + 2y + 10$, when $x = 240$ and $y = 12$.
13. $15m + 3n + 17$, when $m = 112$ and $n = 28$.
14. $2ax + 5x + 4$, when $a = 20$ and $x = 12$.
15. $4xy + 2y + 8$, when $x = 3$ and $y = 12$.
16. $3mn + 15n + 2$, when $m = 20$ and $n = 4$.
17. $6pq + 3q + 21$, when $p = 4$ and $q = 28$.
18. $8rs + 15s + 45$, when $r = 24$ and $s = 60$.

EXERCISE 20.—BRACKETS II

1. $10 + 5 + 3 - 1 =$.
2. $10 + (5 + 3) - 1 =$.
3. $10 + 5 + (3 - 1) =$.
4. $10 + (5 + 3 - 1) =$.
5. $10 - 5 - 3 + 1 =$.
6. $10 - (5 - 3) + 1 =$.
7. $10 - 5 - (3 + 1) =$.
8. $10 - (5 - 3 + 1) =$.
9. Look at the sign before the brackets and say : (*a*) how examples 2, 3, and 4 above are alike, and (*b*) how 6, 7, and 8 are alike.
10. If I removed the brackets in 2, 3, and 4, and worked the examples like example 1, what difference would it make in the result ?
11. If I removed the brackets in 6, 7, and 8, and worked the examples like 5, would the results still be the same ?
12. (*a*) $12 - (4 + 3 + 2) =$.
    (*b*) $12 - 4 - 3 - 2 =$.
13. (*a*) $5 - (4 - 3 + 2) =$.
    (*b*) $5 - 4 + 3 - 2 =$.
14. If minus comes before brackets, I can remove the brackets so long as I change all the . . . within the brackets. Supply the missing word.

Evaluate each of the following in two ways : (i) by first simplifying the expression within the brackets ; (ii) by first removing the brackets and, when necessary, changing the signs. The two results should, of course, be the same.

15. $4 + 8 + (16 + 3 + 5)$.
16. $5 + 9 - (2 + 7 + 3)$.
17. $3 + 8 - (12 - 11)$.
18. $4 + (6 + 7) - (6 + 7)$.
19. $(3 + 5 - 2) - (7 - 3)$.
20. $6 + (5 - 4) - (4 - 1) - 4$.

If brackets are included in larger brackets, such as [ ] or { }, it does not matter whether you remove the smaller brackets first or the larger ones. One method may be used to check the other.

21. $7 - [6 - (4 - 2)]$.
22. $4 + 9 - \{8 - (5 + 2)\}$.
23. $9 - 3 - [4 - (2 + 1)]$.
24. $5 + (8 - 2) - [7 - (4 - 1)]$.

EXERCISE 21.—SUBTRACTION

Compare these two subtraction sums :

8,	9	7	6	$8x^3 + 9x^2 + 7x + 6$
2,	5	0	1	$2x^3 + 5x^2 \qquad + 1$
6,	4	7	5	$6x^3 + 4x^2 + 7x + 5$

If x is 10, then the two sums are exactly the same. The second sum can be put like this :

$$8x^3 + 9x^2 + 7x + 6 - (2x^3 + 5x^2 + 1)$$
$$= 8x^3 + 9x^2 + 7x + 6 - 2x^3 - 5x^2 - 1.$$

Work the following examples :

1. $5x^2 + 8x + 3 - (2x^2 + 7x + 1)$.
2. $x^2 + 6x + 5 - (x^2 + 2x + 4)$.
3. $3x^3 + 8x^2 + 5x + 6 - (x^3 + 7x^2 + 4x + 2)$.
4. $8x^3 + 5x^2 + 6x + 3 - (x^3 + 4x^2 + 6x + 2)$.
5. $4x^3 + 8x^2 + 3x + 7 - (x^3 + 4x^2 + 6)$.
6. $3x^3 + 9x^2 + 2x + 7 - (4x^2 + x)$.
7. $5x^4 + 8x^3 + 9x^2 + 3x + 7 - (x^4 + 4x^3 + 9x^2 + 6)$.
8. $3x^4 + 7x^3 + x + 6 - (7x^3 + 4)$.

The first parts of these expressions might have been enclosed in brackets too; but there is no point in enclosing them, as they are the same whether the brackets are there or not.

The term "sum" in algebra includes subtraction as well as addition. The algebraic sum of $5a + 7a - 3a = 9a$.

Find the algebraic sum of the following :

9. $8p - 4p + 2p + 5p - 6p$.
10. $5a + 6b - 4a - 5b + 3a + b$.
11. $3x + 8y - x + 2y - 2x + 7y$.
12. $7m^2 + 6m - 3m^2 + m - m^2 - 7m$.

Find the algebraic sum of the following without removing the brackets :

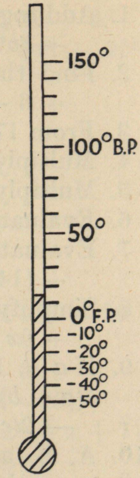
13. $4(a + b) + 3(a + b) + (a + b)$.
14. $5(x - y) - 2(x - y) - 3(x - y)$.
15. $2(m + n - p) + 6(m + n - p) - 5(m + n - p)$.
16. $3(m + n) - 2(m + n) + 6(m + n) - 4(m + n)$.
17. $5(a + b) + 3(a - b) - 4(a + b) - 2(a - b)$.

1. Add together $3x^2 + 4x + 5$, $x^2 + 9x + 7$, $3x + 4$.
2. Add together $a^3 + 3a^2b + 3ab^2 + b^3$, $2a^3 + 2b^3$, $a^2b + ab^2$.
3. Find the algebraic sum of—
   $2x^3 + 7x^2 - x^3 + 3x^2 + 4 - 2x^2 + 6x$.
4. From $5a^3 + 7a^2b + 9ab^2 + 6b^3$
   take $2a^3 + 3a^2b + 7ab^2 + b^3$.
5. Multiply $4n^2 + 8n + 15$ by 5.
6. Multiply $5n^2 + 7n + 3$ by $2n$.
7. Multiply $a^2 + 3a + 9$ by $a + 4$.
8. What money does this expression represent if $x = 20$?
   $5x + 15$.
9. What is the value of $8 + (3 - 2) - (4 + 3)$?
10. Evaluate $15 - (12 - 5) - (11 - 4) - (6 - 5)$.
11. Simplify $5x^3 + 8x^2 + 7x + 9 - (2x^3 + 4x^2 + x + 7)$.
12. If $x = 5$, $y = 3$, and $z = 0$, find the value of—
    $x^2 - (xy - z^2) + 3xy - (xz + yz)$.
13. If the side of a square is $(a + b)$ in. long, what is its perimeter, that is, the distance round it ?
14. If two angles of a triangle are m degrees and n degrees, how many degrees is the third angle ?
15. Write five consecutive numbers, the middle one of which is 8. What is their sum ?
16. Write five consecutive numbers, the middle one of which is n. What is their sum ?
17. If you were x years old 2 years ago, how old will you be in 5 years' time ?
18. What number must be taken from $x + 3y$ to leave $x + y$?
19. It is now x minutes to ten. How many minutes is it past nine ?
20. It is now x minutes past two. How many minutes is it to three ?
21. What must be multiplied: (i) by 4 to make 20 ; (ii) by 5 to make 6 ; (iii) by x to make y ?
22. How many books at p pence each can I buy for £1 ? How many for q pence ?

1. Add together $a^4 + 7a^3b + 3ab^3 + 5b^4$, $4a^3b + 5a^2b^2 + 8b^4$, $12a^4 + 7a^2b^2 + 5b^4$.
2. Find the algebraic sum of $8a^3 + 17a^2 - 2a^2 + 12a + 6 - 4a^3 - 3a + a^2 - 3a^3$.
3. From $17y^3 + 6y^2 + 9y + 4$ take $12y^3 + 3y + 4$.
4. Multiply $2x^2 + 5x + 3$ by $6x$.
5. Multiply $5a^3 + 3a^2 + 7a + 3$ by $2a + 3$.
6. Evaluate $25 - (12 + 3) + 4 - (15 - 11) - (5 + 3)$.
7. Evaluate $(13 + 7) - (13 - 7) + (8 + 3) - (8 - 3) - (14 + 6)$.
8. Simplify $5x^4 + 8x^3 + 3x^2 + 7x + 9 - (8x^3 + x^2 + 6x + 5)$.
9. If $a = 1$, $b = 2$, $c = 3$, $d = 0$, find the value of: $4(a + b) - 3(b - a) + 2(b + c) - 4(c - b) + 3(c + d) - 2(c - d)$.
10. A, B, and C are three milestones in line with one another. If the distance from A to B is $(m + n)$ miles and the distance from B to C is $(n + p)$ miles, how many miles is A from C ?
11. What is the nth number of this series : 6, 12, 18, 24, 30, etc. ?
12. If you were p years old q years ago, how old will you be r years hence ?
13. Four boys have $(a + b)$ shillings, $(a - b)$ shillings, $(b + c)$ shillings, and $(b - c)$ shillings respectively. How many shillings have they altogether ?
14. Find the algebraic sum of:

$$\frac{x}{2x - y} + 3\left(\frac{x}{2x - y}\right) - 2\left(\frac{x}{2x - y}\right) + 7\left(\frac{x}{2x - y}\right).$$

15. What is the average of $3a$, $4a$, and $8a$?
16. A school has p departments. In each department there are q classes. In each class there are r children. How many children are there in the school ?
17. How far will a motor-car going at the rate of x feet per second go in y seconds if it stops z seconds for refilling ?



A Centigrade thermometer has 100 degrees or steps between the freezing-point (0°) and the boiling-point (100°). The degrees below the freezing-point have the minus sign in front of them.

If I call the direction up the thermometer positive or +, I must call the direction down the thermometer negative or —.

Suppose the temperature is exactly 0°, and the mercury rises 7° and then falls 3°, what will be the temperature marked? Obviously 7° — 3° = 4°.

Suppose that starting from 0°, it rose 3° and then fell 7°, what then would be the temperature? The answer is 3 — 7 = — 4. Three up and seven down is the same as four down.

If we take — to mean "take away," 3 — 7 makes nonsense. We cannot take 7 from 3. But if 3 — 7 (or more fully + 3 — 7) means 3 steps in one direction and 7 steps in the opposite direction, 3 — 7 is not nonsense.

Now, suppose the mercury starting at 12° goes 7° up, then 2° down, then 5° up, then 15° down, then another 10° down, where would the thermometer then stand? We can put it this way:

$$+ 12 + 7 - 2 + 5 - 15 - 10.$$

The positive terms: $+ 12 + 7 + 5 = + 24$.

The negative terms: $- 2 - 15 - 10 = - 27$.

$$+ 24 - 27 = - 3.$$

The mercury will stand at — 3°, or 3° below the freezing-point.

Now answer these questions about the thermometer. Name the mark at which the mercury finally stands.

1. Starts at + 4°, up 8°, up 3°, down 7°, up 2°, down 12°, down 5°.
2. Starts at 0°, up 3°, down 5°, down 2°, up 4°, down 6°, up 20°.

3. Starts at $-10°$, up $4°$, down $2°$, up $7°$, down $9°$, up $3°$, down $11°$.

Answer the following questions about a lift in a block of city offices. There are 7 floors, named respectively basement, ground floor, 1st floor, etc., up to 5th floor. The floors are 12 ft. apart. You have to say at which floor the lift has come to rest.

4. Starts at ground floor, up 24 ft., up 12 ft., down 24 ft.
5. Starts at basement, up 60 ft., down 36 ft., up 12 ft.
6. Starts at 5th floor, down 72 ft., up 48 ft., down 36 ft.

A ship is stationed at P. The direction eastward is regarded as positive and the direction westward as negative. State its final position east or west of P. In each case it starts from P.

7. $+ 5$ miles $- 3$ miles $+ 8$ miles $- 15$ miles.
8. $- 10$ miles $- 4$ miles $+ 20$ miles $- 2$ miles.
9. $+ 3$ miles $- 12$ miles $+ 4$ miles $- 8$ miles.

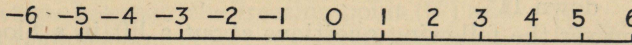
A gambler's winnings are marked positive and his losses negative. State the amount of money he has in hand, or the amount of his debt, in each case.

10. Starts with £10, + £3, − £2, + £4, − £20.
11. Starts with £5, − £2, + £7, − £2, + £6.
12. Starts with £20, − £10, + £5, + £8, − £15, − £10, − £5.

A man has an erratic watch which gains on some days and loses on others. These are the gains and losses for one week. Supposing he has the correct time at the beginning of the week, say by how much his watch is fast or slow at the end of the week.

13. $+ 3$ min. $- 2$ min. $+ 1$ min. $- 4$ min. $+ 2$ min. $+ 1$ min. $- 3$ min.
14. $- 5$ min. $+ 7$ min. $- 4$ min. $+ 6$ min. $- 3$ min. $- 2$ min. $- 6$ min.
15. $+ 1$ min. $+ 9$ min. $- 6$ min. $+ 4$ min. $- 7$ min. $+ 8$ min. $- 1$ min.
16. $- 2$ min. $+ 3$ min. $+ 8$ min. $- 5$ min. $- 7$ min. $- 9$ min. $+ 4$ min.

EXERCISE 23.—POSITIVE AND NEGATIVE NUMBERS



Starting from 0, movement to the right will be taken as forwards or positive, movement to the left as backwards or negative.

Example: Express as positive or negative numbers 3 steps forwards and 5 steps backwards, and state the final position.

$$3 - 5 = -2.$$

Do the same with these :

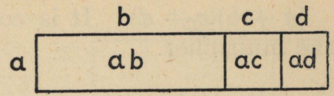
1. 8 forwards and 2 backwards.
2. 5 forwards and 7 backwards.
3. 3 backwards and 4 backwards.
4. 4 forwards, 8 forwards, 7 backwards, 3 backwards.
5. 2 forwards, 12 backwards, 7 backwards, 8 forwards.

Express these in one term :

6. $+ 4 - 7 + 3 - 8 + 2$.
7. $- 3 + 4 - 6 - 5 + 14 - 7$.
8. $+ 14 - 16 - 12 - 10 + 27 - 23$.
9. $+ a - 2a$.
10. $- 5x + 2x + x$.
11. $+ 4y + 3y - 2y - y$.
12. $+ 3x^2 + 7x^2 - 5x^2 + 2x^2 - 6x^2$.
13. $- 4ab + 15ab - 12ab + 27ab$.
14. $+ 12a^2y - 10a^2y + 3a^2y + 8a^2y - 15a^2y$.
15. $+ 4\sqrt{x} + 3\sqrt{x} - 7\sqrt{x} - 3\sqrt{x} + \sqrt{x} - 3\sqrt{x}$.

In an arithmetical expression, such as 374, all the signs are positive ; but in an algebraic expression, such as $3x^2 - 7x + 4$, we may have either positive or negative terms. If we have to subtract such an expression, all we have to do is to change all the signs and then find the algebraic sum.

16. From $9x^3 - 3x^2 + 5x - 2$ take $2x^3 + 5x^2 - 3x - 4$.
17. From $2x^3 - y^3$ take $x^3 - 3x^2y + 3xy^2 - y^3$.



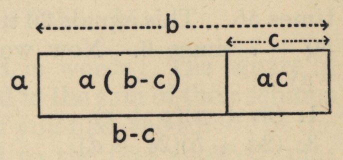
Look at this rectangle. Its length is $b + c + d$ and its breadth is a. Its area is therefore $a(b + c + d)$. But this is made up of the three areas ab, ac, and ad.

Therefore $\mathbf{a(b + c + d) = ab + ac + ad}$.

Testing this by substituting numbers, such as $a = 2$, $b = 5$, $c = 4$, and $d = 3$, we find that the left-hand expression $= 2(5 + 4 + 3) = 2 \times 12 = 24$.

The right-hand expression $= 2 \times 5 + 2 \times 4 + 2 \times 3 = 10 + 8 + 6 = 24$.

Let us now see whether $a(b - c) = ab - ac$.



The area of the whole rectangle in the accompanying figure is ab; and the area of the whole rectangle minus the rectangle to the right is $a(b - c)$.

Therefore $a(b - c) = ab - ac$.

Testing this by substituting 3 for a, 5 for b, and 2 for c, we find:

The left side $= 3(5 - 2) = 3 \times 3 = 9$.

The right side $= 3 \times 5 - 3 \times 2 = 15 - 6 = 9$.

Write the following expressions with the brackets removed:

1. $m(m + n)$.
2. $m^2(m - n)$.
3. $x(x + y + z)$.
4. $ax(x - y - z)$.
5. $a(a^2 + 2a + 1)$.
6. $ab(3a^3b - a^2 + 5a - 4)$.
7. $2(2a + b + 3c)$.
8. $2a(7a - 3b - c)$.
9. $3x(x^2 + xy + y)$.
10. $2xy(x^2y^2 - xy - z)$.

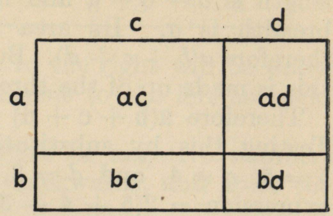
By using brackets express each of the following expressions as a term of two factors:

11. $ax + ay$.
12. $a^2x - a^2y$.
13. $bx + by + bz$.
14. $abx - aby + abz$.
15. $ca^3 + ca^2 + ca$.
16. $3a^2b - 3ab^2 + 6ab$.
17. $4x^2 + 8xy + 6xz$.
18. $14m^2n + 10mn^2 - 6mn$.

EXERCISE 25.—MULTIPLICATION IV

$(a + b)(c + d)$. It is convenient to regard $a + b$ as the multiplier.

$$\begin{array}{l} \;\; c + d \\ \;\; \underline{a + b} \\ ac + ad \\ \qquad\quad \underline{bc + bd} \\ \underline{ac + ad + bc + bd} \end{array}$$



Here is another way of setting it out:
$(a + b)(c + d) = a(c + d) + b(c + d) = ac + ad + bc + bd.$

This is what we do when we find the value of, say, 27 times 56. This equals 20 times 50 + 20 times 6 + 7 times 50 + 7 times 6. Now work the following in the same way:

1. $(m + n)(p + r)$.
2. $(c + d)(e + f)$.
3. $(2a + b)(2c + d)$.
4. $(w + 2x)(2y + z)$.
5. $(2a + 3b)(3c + 4d)$.
6. $(x + 1)(y + 2)$.
7. $(a + b)(b + c)$.
8. $(x + y)(y + 3)$.
9. $(ab + b)(bc + d)$.
10. $(3x + 2y)(x + 2)$.

The same principle applies when there are three or more terms in the multiplicand, e.g.:

$$\begin{aligned}(a + b)(c + d + e) &= a(c + d + e) + b(c + d + e)\\ &= ac + ad + ae + bc + bd + be.\end{aligned}$$

11. $(2a + b)(3c + 4d + e)$.
12. $(x + y)(m + n + p)$.
13. $(x + 2y)(2m + 3n + 4p)$.
14. $(p + q)(5a + 4b + 7c)$.
15. $(m + 3n)(4x + y + 2z)$.
16. $(3a + 4b)(2x + 7y + 3z)$.

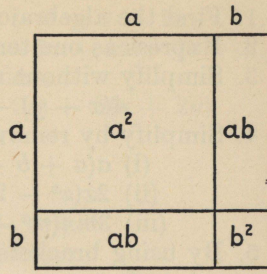
Similarly:

$(a + b + c)(d + e + f) = a(d + e + f) + b(d + e + f) + c(d + e + f) = ad + ae + af + bd + be + bf + cd + ce + cf.$

17. $(p + q + r)(x + y + z)$.
18. $(r + s + t)(x + y + z)$.
19. $(2a + 3b + c)(2p + 3q + 5r)$.
20. $(5m + 2n + p)(x + 2y + 5z)$.
21. $(a + b + c)(d + e + f + g)$.

EXERCISE 26.—$(A + B)^2$

$$
\begin{array}{l}
(a + b)(a + b) = (a + b)^2 \\
\qquad a + b \\
\qquad a + b \\
\hline
\qquad a^2 + ab \\
\qquad\qquad ab + b^2 \\
\hline
\qquad a^2 + 2ab + b^2 \\
\hline
\end{array}
$$



This important statement should be committed to memory :

$$(\mathbf{a} + \mathbf{b})^2 = \mathbf{a}^2 + \mathbf{2ab} + \mathbf{b}^2.$$

It should also be remembered in words : **The square of the sum of two numbers is equal to the sum of their squares plus twice their product.** Or, if you like, the square of the sum of two numbers is equal to the first squared, plus twice their product, plus the second squared.

Suppose I wish to know the value of $(x + 1)^2$, I need not work it out as a multiplication sum, but put down straight off $x^2 +$ (twice $x \times 1$) $+ 1^2$ or $x^2 + 2x + 1$.

Apply the formula $(a + b)^2 = a^2 + 2ab + b^2$ to expand the following :

1. $(m + n)^2$.	2. $(ab + c)^2$.	3. $(2a + b)^2$.
4. $(a + 2b)^2$.	5. $(ab + cd)^2$.	6. $(2a + 2b)^2$.
7. $(7 + 8)^2$.	8. $(8 + 9)^2$.	9. $(9 + 10)^2$.
10. $(5x + 6y)^2$.	11. $(4x + 3)^3$.	12. $(40 + 60)^2$.
13. $(x^2 + y^2)^2$.	14. $(2x^2 + y)^2$.	15. $(x + 2y^2)^2$.

Apply the formula $a^2 + 2ab + b^2 = (a + b)^2$ to writing the following expressions as squares :

16. $p^2 + 2pr + r^2$.	17. $x^2 + 2xy + y^2$.
18. $4m^2 + 12mn + 9n^2$.	19. $2^2 + 2 \times 2 \times 3 + 3^2$.
20. $10^2 + 2 \times 10 \times 20 + 20^2$.	21. $4c^2 + 4cd + d^2$.
22. $a^4 + 2a^2b^2 + b^4$.	23. $x^6 + 2x^3y^3 + y^6$.
24. $9r^2 + 24rs + 16s^2$.	25. $16p^4 + 24p^2q + 9q^2$.

1. Find the algebraic sum of 3, -7, 5, -8.
2. Express as one term $18a + 3a - 20a - 6a + a$.
3. Simplify without removing brackets :
$$4(x + y) - 3(x + y) - 2(x + y).$$
4. Simplify by removing brackets :
   (i) $a(a + b - c)$.
   (ii) $2x(x^2 - 2xy + y^2)$.
   (iii) $3mn(m^3 + 2m^2n - 3mn^2 + n^3)$.
5. By using brackets, resolve the following expressions into factors :
   (i) $bc + bd$.
   (ii) $dfg^2 - d^2fg + df^2g - d^2f^2$.
   (iii) $4x^2y^2 - 8xy^3 + 6x^3y$.
6. Work out the values of the following products :
   (i) $(w + x)(y + z)$.
   (ii) $(a + 2)(a + 3)$.
   (iii) $(x + 2y)(y + 3z)$.
7. Expand the following :
   (i) $(p + q)^2$.
   (ii) $(x + 2)^2$.
   (iii) $(3m + n)^2$.
8. Write the following as squares :
   (i) $4a^2 + 4ab + b^2$.
   (ii) $a^2 + 4ab + 4b^2$.
   (iii) $9x^2 + 12xy + 4y^2$.
9. What is the square root of $y^2 + 2y + 1$?
10. Simplify $5x^2 - 3x - (2x^2 - 3x)$.
11. When $a = 3$, $b = 2$, and $c = 0$, find the value of—
$$a^2 - ab + \tfrac{1}{3}a^2 - bc + c^2.$$
12. What is the sum of the coefficients in the expression :
$$5x^3 + 2x^2 + x \,?$$
13. Find the sum of $2a^2 - 2ab + 3b^2$, $-a^2 + 4ab - 2b^2$, and $5a^2 - 2ab + b^2$.
14. From $x^4 - 5x^3 + 3x - 2$ take $x^4 + 5x^3 - 3x - 2$.
15. From the sum of $3a^2 - 4ab + 3b^2$ and $5a^2 - 7b^2$ take $4a^2 - 4ab$.
16. Take $4ab$ from $(a + b)^2$.

1. Find the algebraic sum of $6 - 7 + 8 - 9 + 10 - 11$.
2. Express as one term: $12a - 16a - 5a + 4a + 7a - a$.
3. Simplify without removing brackets :
   $3(a + 2y) + 9(a + 2y) - 4(a + 2y) - 5(a + 2y)$.
4. Simplify by removing brackets :
   (i) $m^2(2m^2 + 3mn - 5n^2)$.
   (ii) $3xy(2x^2 - 7xy + 3y^2)$.
   (iii) $2p^2(p^3 - 4p^2q + 3pq^2 - 5q^3)$.
5. By using brackets resolve into factors :
   (i) $a^3b - a^2b^2 + ab^3$.
   (ii) $9x^5 - 12x^4y + 3x^3y^2 - 6x^2y^3$.
   (iii) $2x^5y - 8x^4y^2 + 6x^3y^3 - 10x^2y^4 + 4xy^5$.
6. Work out the values of the following products :
   (i) $(a + b)(x + y)$.
   (ii) $(3x + 5)(2x + 3)$.
   (iii) $(2a + 3b)(3x + 2y)$.
7. Expand the following :
   (i) $(p + 2q)^2$.
   (ii) $(3x + 4y)^2$.
   (iii) $(\frac{1}{2}a + 1)^2$.
8. Write the following as squares :
   (i) $4a^2 + 12ab + 9b^2$.
   (ii) $16b^2 + 24bc + 9c^2$.
   (iii) $9x^2 + 30xy + 25y^2$.
9. What is the square root of $9x^2 + 6x + 1$?
10. Simplify $3x^2 - (4x^2 - x) - (5x - 3) + 3$.
11. When $a = 4$, $b = 2$, and $c = 0$, find the value of—
    $a^3 + 3a^2c - \frac{3}{2}b^3 + bc^2 - 3a^2b + 2ab^2$.
12. What is the sum of the indices in—
    $5x^4 - 3x^3 + 2x^2 - 4x + 6$?
13. Add together $6a^4 + 3a^3b - 5a^2b^2 + 6ab^3 - 4b^4$ and $-6a^4 + a^3b - 4a^2b^2 + 6ab^3 + 4b^4$.
14. Take the second expression in question 13 from the first.
15. Take any two numbers. (i) First add them and then square the sum. (ii) First square them and then add the squares. Which of these two procedures gives the larger result, and by how much ?

EXERCISE 27.—THE RULE OF SIGNS

$-$  $+$

When the motor-car moves towards $+$ it is going in a positive direction; when it turns round and moves towards $-$ it is going in a negative direction.

The car travels at 30 miles an hour for 4 hours. We find the distance travelled by multiplying 30 miles by 4. The number 4 is the multiplier. When a forward gear is used the multiplier is positive; when a reverse gear is used the multiplier is negative.

When heading towards $+$ on a forward gear it travels in miles $(+30) \times (+4) = +120$.

When heading towards $-$ on a forward gear it travels in miles $(-30) \times (+4) = -120$.

When heading towards $+$ on the reverse gear it travels in miles $(+30) \times (-4) = -120$.

When heading towards $-$ on the reverse gear it travels in miles $(-30) \times (-4) = +120$.

Hence the rule of signs: **Like signs give plus; unlike signs give minus.** When no sign is used *plus* is understood. Thus $30 \times 4 = (+30) \times (+4)$ and—

$$ab = (+a)(+b).$$

Multiply:

1. m by n. 2. $-m$ by n. 3. m by $-n$.
4. $-m$ by $-n$. 5. $-ab$ by $-ab$. 6. $3xy$ by $-2x$.
7. $-xy$ by -6. 8. pr^2 by $-2p$. 9. $-a^2b^2$ by $3a^2b^2$.
10. $a+b$ by a. 11. $a+b$ by $-a$. 12. $a-b$ by $-a$.
13. $a-b+c$ by $-a$. 14. $a+b-c$ by $-b$.
15. $a-b-c$ by $-c$.

Express the following without brackets:

16. $3(m+n-p)$. 17. $-3(m+n-p)$.
18. $-2(2x^2+3x-1)$. 19. $ab(a-b+c)$.
20. $-ab(a-b+c)$. 21. $-3a(a^2+3ab+2b^2)$.
22. $-x(x^3+2x^2-3x-5)$.
23. $-2y(x^3-2x^2y-3xy^2+y^3)$.
24. $-2m^2n(m^3+3m^2n-3mn^2-n^3)$.

EXERCISE 28.—BRACKETS III

The rule of signs applies to the sign preceding the brackets and the signs within the brackets, e.g. :

$$a + (b - c) = a + b - c.$$
$$a - (b - c) = a - b + c.$$

This truth can be tested by substituting numbers for a, b, and c. The $+$ before the bracket presents no difficulty. The brackets can be removed without any change of sign in the numbers inside the brackets.

But when $-$ is before the brackets the brackets must not be removed without changing all the signs within the brackets.

It is easy to see that $a - (b + c) = a - b - c$, because both the b and the c have to be taken away. In fact, $-(+c) = -c$. If $-(+c) = -c$, then $-(-c)$ must have the opposite value, viz. $+c$.

Test, by substituting 6 for a, 4 for b, 2 for c, and 1 for d, whether these identities are correct, and write down what each side of the identity comes to :

1. $a + (b + c) = a + b + c.$
2. $a + (b - c) = a + b - c.$
3. $a - (b + c) = a - b - c.$
4. $a - (b - c) = a - b + c.$
5. $2a - (b + c + d) = 2a - b - c - d.$
6. $3a(b + 2c) - 2a(b - 2c) = 3ab + 6ac - 2ab + 4ac = ab + 10ac.$
7. $a + 2b - c = a + (2b - c).$
8. $a - b + 2c = a - (b - 2c).$
9. $a - b + c - d = a - (b - c + d).$
10. $3a - 2b - 2c - 2d = 3a - 2(b + c + d).$

Remember : $-(+a) = -a$, and $-(-a) = a$.

When removing brackets preceded by a $+$ sign, leave the signs inside the brackets unchanged.

When removing brackets preceded by a $-$ sign, change all the signs inside the brackets.

EXERCISE 29.—POSITIVE AND NEGATIVE PARCELS

We have already learnt that when we put terms within brackets we make a parcel of them and wish them to be treated as a single term or a single number.

$$\text{(i) } x + (a + b).$$
$$\text{(ii) } x - (a + b).$$

In these two cases $a + b$ has been tied up in a parcel. But there is a great difference between these two parcels. One is a positive parcel and the other a negative. The terms in a positive parcel are the same when they are in the parcel as they are when the parcel is untied and they are taken out. In other words, you can rub the brackets out without its making any difference, or you can put them in without its making any difference. This is not true of the negative parcels. Terms inside a negative parcel have the opposite signs of what they have outside. So you cannot put terms in a negative parcel without changing all the signs ; and you have to change them all back again when you take them out.

Note that a vinculum binds just as brackets do :

$$\frac{a + b}{a - b} \text{ is the same as } (a + b) \div (a - b).$$

In each of the following expressions tie the 2nd and 3rd terms into a positive parcel, and the 4th, 5th, and 6th terms into a negative parcel :

1. $4x^5 + x^4 + 2x^3 - 2x^2 + x - 3.$
2. $a^5 + 3a^4 - 4a^3 - 3a^2 + 5a + 7.$
3. $3a^5 + a^4y - 3a^3y^2 - 4a^2y^3 - 2ay^4 + y^5.$
4. $7n + 2n + 5p - m - 3n + 5p.$
5. $2x^2 + 5p - 2q - 4p^2 - pq - 2r^2.$
6. $5x^5 + x^4y + 4x^3y^2 - 7x^2y^3 - 4xy^4 - 3y^5.$

Untie these parcels ; that is, remove the brackets :

7. $(3x^4 - 2x^3) + (5x^2 - 6x - 4).$
8. $(5x^4 + 2x^3) - (3x^2 + 5x - 2).$
9. $3a^5 - (2a^4b + 5a^3b^2) - (a^2b^3 - 3ab^4 + 4b^5).$

Note that in the expression $x - 2(a - b)$ the second term consists of two factors, 2 being one and $(a - b)$ the other. We know that $2(a - b) = (2a - 2b)$. This can be verified by substituting numbers for a and b.

$-2(a - b) = -2a + 2b$. I can regard this process as multiplying each term within the brackets by -2 and at the same time removing the brackets, or as changing the signs because of the minus before the brackets, and then multiplying by 2. It comes to the same thing.

I multiply the following expression by 6 in this way :

$$\frac{2x-3}{2} - \frac{3x-2}{3}$$

Multiply by 6 :

$3(2x - 3) - 2(3x - 2) = 6x - 9 - 6x + 4 = -5.$

Multiply each of the following by 12 :

10. $\dfrac{3a+4}{2} - \dfrac{2a-3}{6} + \dfrac{4a-5}{4}.$

11. $\dfrac{5n-2}{3} - \dfrac{2n-3}{4} - \dfrac{n-5}{2}.$

12. $\dfrac{6p-1}{6} + \dfrac{3p+4}{4} - \dfrac{2p-3}{3}.$

Look at the expression $5a^4 - 3a^3b + 6ab^3$. I can bracket the last two terms thus : $5a^4 - (3a^3b - 6ab^3)$. The expression in the brackets can be resolved into the factors $3ab(a^2 - b^2)$. Therefore the whole expression may be written $5a^4 - 3ab(a^2 - b^2)$. In the following expressions put the first two terms into positive parcels and the last two into negative parcels. Then factorize where possible.

13. $3a + 3b - 4c - 4d.$
14. $2x + 4y - 3z + 12w.$
15. $a^2 + ab - bc + bd.$
16. $3a^2b - 9abc - 2a^2c - 6acd.$
17. $mn - mp - np - nq.$
18. $5p^2q - 10pq^2 - pqr + qr^2.$
19. $3st^2 + 12s^2t - st - t^2.$
20. $x^4 - x^2 - 4x + 8.$
21. $c^4d - cd^4 - 2c^3d^2 + 6c^2d^3.$
22. $t^6 + t^3 - 3t^2 - 9.$
23. $4x^2 - 6xy - 6xy + 9y^2.$
24. $15a^3b - 10ab^3 - 3a^3 + 2ab^2.$

EXERCISE 30.—SUBSTITUTION OF NEGATIVE NUMBERS

When no sign is placed before a term, + is understood.

$$a \times b = ab \qquad \frac{-a}{b} = -\frac{a}{b}$$

$$a \times (-b) = -ab \qquad \frac{a}{-b} = -\frac{a}{b}$$

$$(-a) \times b = -ab \qquad \frac{-a}{-b} = \frac{a}{b}$$

$$(-a) \times (-b) = ab$$

The above examples illustrate the rule of signs: **Like signs give plus; unlike signs give minus.** This rule applies to both multiplication and division. Suppose I have to substitute -2 for x in the following expression:

$$5x^2 - 3(x + 1) + 2.$$

I can begin by substituting at once:

$$5(-2)(-2) - 3\ \{(-2) + 1\}\ + 2$$
$$= 20 - 3(-1) + 2 = 20 + 3 + 2 = 25.$$

Or, I can simplify first:

$$5x^2 - 3x - 3 + 2 = 5x^2 - 3x - 1$$
$$= 5(-2)(-2) - 3(-2) - 1 = 20 + 6 - 1 = 25.$$

When there are two ways of working an example, one way should be used to check the other.

Find the value of:

1. $3x^2 - 5x + 6$ when $x = -3$.
2. $4(x - 2) - 5(x - 4)$ when $x = -1$.
3. $3(x + 4) + 4(x - 2) - (x + 5)$ when $x = -2$.
4. $x^3 - 2x^2 + 5x - 3$ when $x = -3$.
5. $p^4 - p^3 + p^2 - p + 1$ when $x = -2$.
6. $15(n - 1) - 12(n + 2) + 16n$, when $n = -5$.
7. $\dfrac{n+2}{n-5} + \dfrac{n-5}{n+2} - \dfrac{n+7}{n-2}$ when $n = -1$.
8. $\dfrac{p^2-1}{2} - \dfrac{p+2}{3} + \dfrac{5}{6}$ when $n = -3$.
9. $4(t^2 + 2t + 1) - 3(2t^2 - t - 9)$ when $t = -2$.

EXERCISE 31.—MULTIPLICATION V

$$(2x - 1)(3x - 2)$$

I can simplify this in two ways. One is by working it as in arithmetic :

$$\begin{array}{l} 3x - 2 \\ \underline{2x - 1} \\ 6x^2 - 4x \\ \underline{\quad\;\; - 3x + 2} \\ \underline{6x^2 - 7x + 2} \end{array}$$

The other way is this :

$$(2x - 1)(3x - 2) = 2x(3x - 2) - (3x - 2)$$
$$= 6x^2 - 4x - 3x + 2 = 6x^2 - 7x + 2$$

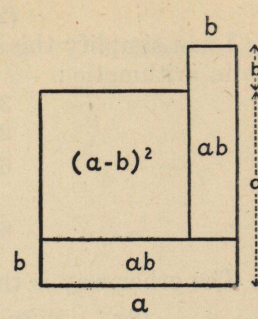
Simplify the following :

1. $(a - b)(c - d)$.
2. $(a + b)(c - d)$.
3. $(a - b)(c + d)$.
4. $(2x - 1)(x + 1)$.
5. $(2x + 1)(x - 1)$.
6. $(m - n)(2m - n)$.
7. $(x - y)(x + 2y)$.
8. $(x - y)(x - 2y)$.
9. $(x^2 - y)(x + y^2)$.
10. $(x^2 - y)(x - y^2)$.
11. $(a^2 + a + 1)(a + 1)$.
12. $(a^2 - a + 1)(a - 1)$.
13. $(x^2 - x + 2)(x + 2)$.
14. $(x^2 + x - 2)(x - 2)$.
15. $(m^2 - m - 1)(m + 1)$.
16. $(m^2 + m - 1)(m - 1)$.
17. $(a^2 + ab + b^2)(a + b)$.
18. $(a^2 - ab + b^2)(a + b)$.
19. $(a^2 - ab + b^2)(a - b)$.
20. $(a^2 + ab + b^2)(a - b)$.
21. $(x^2 + 2xy + y^2)(x + y)$.
22. $(x^2 - 2xy + y^2)(x - y)$.
23. $(a + b - c)(c + d)$.
24. $(a - b + c)(c - d)$.
25. $(m - n + p)(r + s)$.
26. $(m - n + p)(r - s)$.
27. $(n^2 + 1)(n + 1)$.
28. $(n^2 - 1)(n - 1)$.
29. $(a^2 - b^2)(a + b)$.
30. $(a^2 + b^2)(a - b)$.
31. $(7x^2 + 3)(2x - 1)$.
32. $(8x^2 - 4)(x + 2)$.
33. $(a + b)(a + b)(a + b)$.
34. $(a - b)(a - b)(a - b)$.
35. $(a + b)(b + c)(c + d)$.
36. $(a - b)(b - c)(c - d)$.
37. $(x + 1)(x + 2)(x + 3)$.
38. $(x - 1)(x - 2)(x - 3)$.
39. $(2a+b)(3a+b)(4a+b)$.
40. $(2a-b)(3a-b)(4a-b)$.
41. $(m + n)(m - n)$.
42. $(2m + 3n)(2m - 3n)$.
43. $(a + b)(a - b)(a^2 - b^2)$.
44. $(a + b)(a - b)(a^2 + b^2)$.
45. $(x + 1)(x - 1)(x^2 + 1)$.
46. $(y - 1)(y + 1)(y^2 - 1)$.

EXERCISE 32.—$(A - B)^2$

$$(a - b)(a - b) = (a - b)^2$$

$$\begin{array}{rrr} a & - & b \\ a & - & b \\ \hline a^2 & - & ab \\ & - & ab + b^2 \\ \hline a^2 & - & 2ab + b^2 \\ \hline \end{array}$$



The whole of this figure consists of two squares, a^2 and b^2. It illustrates the fact that $(\mathbf{a} - \mathbf{b})^2 = \mathbf{a}^2 - \mathbf{2ab} + \mathbf{b}^2$.

The square of the difference of two numbers is equal to the sum of their squares minus twice their product.

Or, the square of the difference of two numbers is equal to the first squared, minus twice their product, plus the second squared.

Verify the fact that $(a - b)^2 = a^2 - 2ab + b^2$ by making:

1. $a = 3$ and $b = 2$.
2. $a = 9$ and $b = 1$.
3. $a = 100$ and $b = 2$.
4. $a = 1{,}000$ and $b = 3$.

Use the above formula to expand the following:

5. $(x - y)^2$.
6. $(x - 1)^2$.
7. $(2x - 1)^2$.
8. $(2a - b)^2$.
9. $(x^2y^2 - xy)^2$.
10. $(x^3 - 1)^2$.
11. $(10 - 2)^2$.
12. $(100 - 4)^2$.
13. $(3a - 2b)^2$.
14. $(m^2 - n^2)^2$.
15. $(x^2 - y)^2$.
16. $(2y^2 - 3)^2$.

Use the same formula for writing the following expressions as squares:

17. $s^2 - 2st + t^2$.
18. $p^2 - 2pq + q^2$.
19. $4a^2 - 4ab + b^2$.
20. $a^2 - 4ab + 4b^2$.
21. $9x^2 - 18xy + 9y^2$.
22. $x^4 - 2x^2y^2 + y^4$.
23. $9m^4 - 6m^2 + 1$.
24. $4m^4n^4 - 4m^3n^3 + m^2n^2$.
25. $25x^2 - 10x + 1$.
26. $9a^2 - 12ab + 4b^2$.
27. $4m^2 - 20mn + 25n^2$.
28. $x^4 - 8x^2y^2 + 16y^4$.
29. $9p^6 - 24p^3q^3 + 16q^6$.
30. $16s^6 - 24s^3t^3 + 9t^6$.

EASY TEST 6

1. Write out these products: (i) $x(-y)$, (ii) $-2b(a+b)$, (iii) $-p^2(p^2-2pq+q^2)$.
2. Remove brackets and simplify:
   (i) $a^2-(ab-b^2)$.
   (ii) $5(x+1)-3(x-1)-(x+1)$.
   (iii) $3a(a+b)+a^2(c+1)-4(a^2+ab)$.
3. Put every term except the first between brackets in each of the following:
   (i) $a^2-2ab+b^2$.
   (ii) $5x^3-3x^2y+2xy^2-y^3$.
   (iii) $\frac{1}{2}p^3-\frac{1}{4}p^2q+\frac{1}{8}pq^2-\frac{1}{2}q^3$.
4. Resolve the following into two factors, the simpler of them a negative factor:
   (i) $-a^2b-ab^2$.
   (ii) $-3x^3+9x^2-12x$.
   (iii) $-15p^5-5p^4q+25p^3q^2+10p^2q^3$.
5. Multiply the following by 6 and simplify the result:
$$\frac{4a-5}{2}+\frac{3a-4}{3}.$$
6. When $x=-2$, what is the value of x^2-3x+4?
7. When $x=-3$, what is the value of $3x^2-5x-6$?
8. Evaluate the product $(x+y)(x-y)$.
9. Multiply out $(2a-b)(a-2b)$.
10. Expand: (i) $(a-y)^2$, (ii) $(b-1)^2$, (iii) $(2m-n)^2$.
11. When $p=4$, $q=3$, $r=1$, $s=0$, find the value of—
$$2p^2-4ps+pq-3qr+rs.$$
12. Expand (i) $(m-n)^2$, (ii) $(3x-y)^2$.
13. Write as squares:
   (i) $9a^2-24ab+16b^2$.
   (ii) $4m^2-12mn+9n^2$.
14. How much is the square of the difference of d and f?
15. Find the square root of $16c^2-8c+1$.
16. Simplify the following by cancelling:
$$\text{(i) } \frac{2a+2b}{2}; \quad \text{(ii) } \frac{3a+9b}{3}; \quad \text{(iii) } \frac{10x-15y}{5}.$$

1. Write out these products:
   (i) $(-x)(-y)$.
   (ii) $-3a(a+b)$.
   (iii) $-2x^2y(2x^2-3xy+y^2)$.
2. Remove brackets and simplify:
   (i) $4x^2-5-3(x^2-2)$.
   (ii) $3(p+2)-2(p-2)+5(p-1)-4(p+2)$.
   (iii) $a(2a-b)-b(a-2b)+ab(c+2)$.
3. Put the last two terms between brackets with a factor in front:
   (i) $a^2-2ab-4b^2$.
   (ii) $6x^3-5x^2y+15xy^2$.
   (iii) $p^2-\frac{1}{4}pq+\frac{1}{2}q^2$.
4. Resolve the following into two factors, one of them negative:
   (i) $-2pq+4pq^2-6q^3$.
   (ii) $-6x^3y^2-9x^2y^3+3xy^4-12y^5$.
   (iii) $-12abc^2+8ab^2c-4a^2bc+16a^2b^2$.
5. Multiply the following by 12 and simplify the result:

$$\frac{13x+7}{4}-\frac{3x-4}{2}-\frac{5x-3}{3}$$

6. If $a=-3$, what is the value of a^2-5a+6?
7. If $a=-4$, what is the value of $2a^2-7a-8$?
8. Evaluate the product $(2a+b)(3a-b)$.
9. Multiply out $(3x-2y)(2x-3y)$.
10. Expand: (i) $(b-c)^2$, (ii) $(2c-3d)^2$, (iii) $(3d-4f)^2$.
11. Write as squares: (i) $4p^2q^2-4pq+1$,
    (ii) $\frac{1}{4}m^2-\frac{1}{2}mn+\frac{1}{4}n^2$.
12. Write as squares: (i) $25x^2-10x+1$,
    (ii) $25x^2-20x+4$.
13. What is the square root of $36a^2-60ab+25b^2$?
14. By how much is $(a+b)^2$ greater than $(a-b)^2$?
15. What is the sum of $(a+b)^2$ and $(a-b)^2$?
16. Simplify $\dfrac{2x+6y}{2}-\dfrac{3x+6y}{3}$.

EXERCISE 33.—THE DIFFERENCE OF TWO SQUARES I

$$\begin{array}{ll} a + b & \\ a - b & \\ \hline a^2 + ab & \\ \quad - ab & - b^2 \\ \hline a^2 & - b^2 \\ \hline \end{array} \qquad \begin{array}{ll} x + y & \\ x - y & \\ \hline x^2 + xy & \\ \quad - xy & - y^2 \\ \hline x^2 & - y^2 \\ \hline \end{array}$$

$(a + b)(a - b) = a^2 - b^2$ $\qquad$ $(x + y)(x - y) = x^2 - y^2$

The fact that $(\mathbf{a} + \mathbf{b})(\mathbf{a} - \mathbf{b}) = \mathbf{a}^2 - \mathbf{b}^2$ is so important that it should be memorized in words in one of these two forms :

The sum of two numbers multiplied by their difference equals the difference of their squares.

The product of the sum and difference of two numbers equals the difference of their squares.

When reversed, and put as $a^2 - b^2 = (a + b)(a - b)$, it becomes a labour-saving formula. Suppose, for instance, we had to find the value of $538^2 - 532^2$. Let us square and then subtract, as follows :

$$\begin{array}{r} 538 \\ 538 \\ \hline 269{,}0 \\ 16{,}14 \\ 4{,}304 \\ \hline 289{,}444 \end{array} \qquad \begin{array}{r} 532 \\ 532 \\ \hline 266{,}0 \\ 15{,}96 \\ 1{,}064 \\ \hline 283{,}024 \end{array}$$

$$\begin{array}{lr} \text{By subtraction :} & 289{,}444 \\ & 283{,}024 \\ \hline \text{Ans. :} & 6{,}420 \\ \hline \end{array}$$

Now work it by applying the formula :

$$\begin{aligned} 538^2 - 532^2 &= (538 + 532)(538 - 532) \\ &= 1{,}070 \times 6 \\ &= 6{,}420 \end{aligned}$$

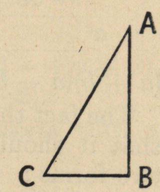
It is obvious that the second method is much easier than the first.

By applying the above formula evaluate the following :

1. $69^2 - 67^2$. 2. $165^2 - 162^2$. 3. $4,287^2 - 4,277^2$.
4. $385^2 - 350^2$. 5. $670^2 - 386^2$. 6. $5,926^2 - 5,382^2$.

It is necessary to make calculations of this kind when dealing with right-angled triangles. We know from our study of geometry that if ABC is a triangle right-angled at B, then—

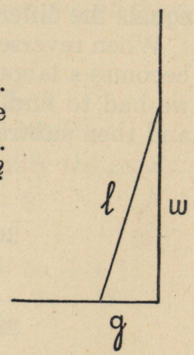
$$AC^2 = AB^2 + BC^2.$$



If a is the number of units of length in the hypotenuse (the side opposite the right angle) and b and c the numbers in the other two sides—

$$a^2 = b^2 + c^2$$
$$\therefore c^2 = a^2 - b^2 = (a + b)(a - b)$$
$$\text{and } c = \sqrt{(a + b)(a - b)}.$$
$$\text{Similarly } b = \sqrt{(a + c)(a - c)}.$$

Consider this problem : A ladder 25 ft. long is placed against a wall so that the foot of the ladder is 5 ft. from the wall. How far up the wall will the ladder reach ?



We have to find w—

$$\begin{aligned} w^2 &= l^2 - g^2 \\ &= (l + g)(l - g) \\ &= (25 + 5)(25 - 5) \\ &= 30 \times 20 \\ &= 600 \\ \therefore w &= \sqrt{600}. \end{aligned}$$

If you know how to find the square root of 600 you will find that it is nearly 24·5. That is to say, the ladder will reach about 24 ft. 6 in. up the wall.

You need not find the square root in answering the following. Give your answer in the form $\sqrt{x}$. In the above diagram find the length in feet of :

7. l when w is 32 ft. and g 6 ft.
8. g when l is 40 ft. and w 39 ft.
9. w when l is 29 ft. and g 4 ft.

EXERCISE 34.—FIRST LAW OF INDICES

The first law of indices is $\mathbf{x^a \times x^b = x^{a+b}}$.

You have already learnt that—

$$x^2 \times x^3 = xxxxx = x^{3+2} = x^5.$$

Similarly $x^5 \times x^6 = x^{5+6} = x^{11}$.

So the general rule is $x^a \times x^b = x^{a+b}$.

x^5 may be called x raised to the 5th power ; and x^a may be called x raised to the ath power. The word "raised" may, if you like, be left out. So we may put the law into words thus : When a number to the ath power is multiplied by the same number to the bth power, the product is that number to the $(a + b)$th power.

In the number x^a the number x is called the base and the number a is called the index (plural : indices). In multiplication, when the base is the same the indices must be added, not multiplied. This does not apply to any other operation except multiplication. We *cannot* say that $x^a + x^b = x^{a+b}$. In fact, $x^a + x^b$ cannot be further simplified. You may test the truth of this by substituting numbers for the letters.

In simplifying the following examples, keep the expressions in brackets as they are. Do not untie the bundles. Remember that $x = x^1$.

1. $a^3 \times a^4$.
2. $m^5 \times m^6$.
3. $p^r \times p^s$.
4. $p \times p^r \times p^s$.
5. $x^2 \times x^3 \times x^5$.
6. $y^3 \times y^5 \times y^6 \times y^8$.
7. $p^a \times p^b \times p^c \times p^d$.
8. $(a + b)^2(a + b)^5$.
9. $(c - d)^3(c - d)^7$.
10. $\left(\frac{x + y}{3}\right)^2\left(\frac{x + y}{3}\right)^3$.
11. $\left(\frac{m + n}{m - n}\right)\left(\frac{m + n}{m - n}\right)^6$.
12. $(\frac{1}{2}x - 5)(\frac{1}{2}x - 5)^3(\frac{1}{2}x - 5)^4$.
13. $(m + n)(m + n)^2(m + n)^3(m + n)^4$.
14. $(x - y)^a (x - y)^b (x - y)^c (x - y)^d$.
15. What is the square of $\left(\frac{x + y}{x - y}\right)^3$?
16. What is the cube of $\left(\frac{x + y}{x - y}\right)^3$?

EXERCISE 35.—SECOND LAW OF INDICES

The second law of indices is $\mathbf{x^a \div x^b = x^{a-b}}$.

You have already learnt that—

$$x^3 \div x^2 = \frac{xxx}{xx} = x^{3-2} = x^1 = x.$$

Similarly $x^5 \div x^2 = x^{5-2} = x^3$.

So the general rule is $x^a \div x^b = x^{a-b}$.

By applying this rule we discover the curious fact that every number, whatever it is, that has 0 as an index $= 1$:

$$1^0 = 5^0 = 27^0 = a^0 = b^0 = x^0 = m^0 = 1.$$

We arrive at this conclusion like this—

$$x^a \div x^a = 1.$$

$$\text{But } x^a \div x^a = x^{a-a} = x^o = 1.$$

(Note : x° means x degrees. The index here is not really nought.)

$$\text{Just as } \frac{6}{2} = 3, \text{ and } \frac{2}{6} = \frac{1}{3},$$

$$\text{so } \frac{x^8}{x^3} = x^5, \text{ and } \frac{x^3}{x^8} = \frac{1}{x^5}.$$

The expression $5x^3 + 3x^2 + 4x + 2$ may therefore be written $5x^3 + 3x^2 + 4x^1 + 2x^0$, and 2 may be regarded as a coefficient of x, just as the other numbers 5, 3, and 4.

Simplify the following, keeping the expressions in brackets as they are :

1. $\frac{x^7}{x^2}$.
2. $\frac{x^2}{x^7}$.
3. $\frac{(a+b)^3}{(a+b)^2}$.
4. $\frac{(n-1)^3}{(n-1)^5}$.
5. $\frac{(2a^2+ab+b^2)^7}{(2a^2+ab+b^2)^3}$.
6. $\frac{8a^3}{2a}$.
7. $\frac{3(5x^2+4x+3)}{9(5x^2+4x+3)^4}$.
8. What is the coefficient of a^0 in this expression—

   $$3a^2 + 7a + 4 \text{ ?}$$

9. Which is the larger, 9^0 or 7^0 ?
10. Add together $a^0 + b^0 + c^0 + d^0 + x^0 + y^0$.

EXERCISE 36.—THE DIFFERENCE OF TWO SQUARES II

Remember : $\mathbf{a}^2 - \mathbf{b}^2 = (\mathbf{a} + \mathbf{b})(\mathbf{a} - \mathbf{b})$.

Factorize $4x^2 - 9y^2$. This is the same as—

$$(2x)^2 - (3y)^2 \text{ and } = (2x + 3y)(2x - 3y).$$

Factorize $(a + b)^2 - (c + d)^2$.

Treat the parts in the brackets as though they were single terms. Indeed, the brackets turn them for the time being into single terms.

$$\begin{aligned}(a + b)^2 - (c + d)^2 &= [(a + b) + (c + d)][(a + b) - (c + d)] \\ &= (a + b + c + d)(a + b - c - d).\end{aligned}$$

Simplify the following by representing them as the difference of two squares :

1. $(m + n)(m - n)$.
2. $(2m + n)(2m - n)$.
3. $(x + 4y)(x - 4y)$.
4. $(3p + 5q)(3p - 5q)$.
5. $(ab + cd)(ab - cd)$.
6. $(\frac{1}{2}v + t)(\frac{1}{2}v - t)$.
7. $(m + n + p + q)(m + n - p - q)$.
8. $(m - n + p - q)(m - n - p + q)$.

Note this example—

$$\begin{aligned}a^4 - b^4 &= (a^2 + b^2)(a^2 - b^2) \\ &= (a^2 + b^2)(a + b)(a - b).\end{aligned}$$

Represent the following as the difference of two terms, each to the fourth power :

9. $(x^2 + y^2)(x + y)(x - y)$.
10. $(9a^2 + 16b^2)(3a + 4b)(3a - 4b)$.
11. $(a^2x^2 + b^2y^2)(ax + by)(ax - by)$.
12. $(\frac{1}{4}p^2 + q^2)(\frac{1}{2}p + q)(\frac{1}{2}p - q)$.

Factorize the following :

13. $s^2 - t^2$.
14. $m^2 - 4n^2$.
15. $9a^2 - 25b^2$.
16. $4w^2x^2 - 9y^2z^2$.
17. $\frac{1}{4}p^2 - \frac{1}{9}q^2$.
18. $\frac{9}{16}c^2 - \frac{16}{25}d^2$.
19. $(r + s)^2 - (t + v)^2$.
20. $(r - s)^2 - (t - v)^2$.
21. $x^4 - y^4$.
22. $16a^4 - 81b^4$.
23. $\frac{1}{16}p^4 - q^4$.
24. $m^4n^4 - p^4q^4$.
25. $\frac{1}{16}x^4 - \frac{1}{81}y^4$.
26. $256x^4 - 625y^4$.

EXERCISE 37.—FACTORS I

Verify these results by multiplying out as rapidly as you can :

$$(a + b)^2 = a^2 + 2ab + b^2.$$
$$(a - b)^2 = a^2 - 2ab + b^2.$$
$$(a + b)(a - b) = a^2 - b^2.$$
$$(x + a)(x + b) = x^2 + (a + b)x + ab.$$
$$(x - a)(x - b) = x^2 - (a + b)x + ab.$$

You should time yourself. If it takes you more than three minutes you should practise them again. To work them over and over again fixes the results on the memory.

Suppose you have to simplify $(a + b)^2 - (a - b)^2$. There are two ways of doing it—

(i) $$(a + b)^2 - (a - b)^2 = (a^2 + 2ab + b^2) - (a^2 - 2ab + b^2)$$
$$= a^2 + 2ab + b^2 - a^2 + 2ab - b^2$$
$$= 4ab.$$

(ii) $(a + b)^2 - (a - b)^2$ being the difference of two squares

$$= [(a + b) + (a - b)][(a + b) - (a - b)]$$
$$= (a + b + a - b)(a + b - a + b)$$
$$= (2a)(2b) = 4ab.$$

Simplify the following :

1. $(x + y)^2 - 2xy.$
2. $(m + n)^2 - (m^2 - n^2).$
3. $(m - n)^2 - (m^2 - n^2).$
4. $(p + q)^2 - (p + q)(p - q).$
5. $p^2 + q^2 - (p + q)^2.$
6. $x^2 + y^2 - (x + y)(x - y).$
7. $(x + 3)^2 - (x + 2)^2.$
8. $(x + 4)^2 - (x + 3)^2.$
9. $(x + 3)^2 - (x - 2)^2.$
10. $(x - 3)^2 + (x - 4)^2.$
11. $(x - 4)^2 - (x + 2)(x + 3).$
12. $(x + 2)(x + 5) - (x - 2)(x - 5).$
13. $(x + t)(x + v).$
14. $(a - b)(a - c).$
15. $(a + 7)(a + 3) + (a - 7)(a - 3).$
16. $(a + 7)(a + 3) - (a - 7)(a - 3).$
17. $(2m + 3)(2m + 4) - (3m + 2)(3m + 4).$
18. $(4x - 1)(4x - 3) - (3x - 1)(3x - 4).$
19. $(2x + y)^2 - (2x - y)^2.$
20. $(3m + 2n)^2 - (3m - 2n)^2.$
21. $(rs + 1)^2 - (rs - 1)^2.$
22. $(a + 5b)^2 - (5a - b)^2.$
23. $(2x + 5y)^2 - (3x - 4y)^2.$
24. $(3x - 4y)^2 - (2x + 5y)^2.$

EXERCISE 38.—SQUARES AND SQUARE ROOTS

Number.		Square.
0	..	0
1	..	1
2	..	4
3	..	9
4	..	16
5	..	25
6	..	36
7	..	49
8	..	64
9	..	81
10	..	100
11	..	121
12	..	144
13	..	169
14	..	196
15	..	225
16	..	256

$3^2 = 3 \times 3 = 9$, and $\sqrt{9} = 3$.
$4^2 = 4 \times 4 = 16$, and $\sqrt{16} = 4$.
$1^2 = 1 \times 1 = 1$, and $\sqrt{1} = 1$.
$x^2 = x \times x$, and $\sqrt{x^2} = x$.

From the table on this page you can find the square of any number up to 16, and approximately the square root of any number up to 256. Thus $\sqrt{225} = 15$. But what is $\sqrt{180}$? It is somewhere between 13 and 14. It is in fact 13 and a fraction.

When we square numbers which have indices, do we square the indices, or do we double them? Is $(2^3)^2 = 2^9$, or is it $= 2^6$? $(2^3)^2 = 2^3 \times 2^3 = 2^{3+3} = 2^6$.

Therefore when we square numbers with indices we double the indices; and when we extract the square root of numbers with indices we halve the indices.

When we square a term consisting of factors, do we square every factor? Is $(abc)^2 = abc^2$, or is it $= a^2b^2c^2$? $(abc)^2 = abcabc = a^2b^2c^2$. We therefore square each factor.

Note that the square of a fraction is less than the fraction itself. For instance, $(\frac{1}{2})^2 = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$, and $\frac{1}{4}$ is less than $\frac{1}{2}$. Similarly, $(\frac{3}{4})^2 = \frac{3}{4} \times \frac{3}{4} = \frac{9}{16}$, and $\frac{9}{16}$ is less than $\frac{3}{4}$.

Keeping brackets undisturbed, write out the squares of the following:

1. 12.	2. 16.	3. m.	4. m^2.
5. m^3.	6. m^{15}.	7. m^x.	8. 1^5.
9. 1^n.	10. xy.	11. x^2y.	12. x^2y^2z.
13. ab^2c^3.	14. $3a$.	15. $12a^3b^3$.	16. $16x^2y^2z^4$.
17. $(a + b)$.	18. $(a + b)^2$.	19. $(a + b)^5$.	20. $5(a - b)^3$.

21. $\left(\frac{x+1}{2}\right)^2$. 22. $\frac{x-1}{2}$. 23. $(a^2+ab+b^2)^4$.

24. $4(x^2+x-1)$. 25. $\frac{1}{3}$. 26. $\frac{4}{5}$.

27. $\frac{12}{13}$. 28. $\frac{1}{2}(x-y)^3$. 29. $\frac{3}{4}\left(\frac{m+n}{m-n}\right)^5$.

30. $(a^2-ab+b^2)^x$.

Extract the square root of each of the following :

31. 81. 32. 121. 33. $\frac{64}{121}$.

34. x^2. 35. x^4. 36. x^8.

37. x^{2m}. 38. x^{6m}. 39. $(a+b)^6$.

40. $4(a+b-c)^4$. 41. $x^2y^4z^6$. 42. $169\left(\frac{a+b}{a-b}\right)^{10}$.

43. $\frac{1}{9}\left(\frac{m-n}{2}\right)^6$. 44. $(p+q)^{4x}$.

Square the following expressions :

45. $c+d$. 46. c^2+d^2. 47. c^3+d^3. 48. $\frac{1}{2}c+\frac{1}{2}d$.
49. $e-f$. 50. e^3-f^3. 51. $\frac{1}{2}e-\frac{1}{2}f$. 52. $2p+q$.
53. $p+2q$. 54. $2p+3q$. 55. $3p+2q$. 56. $ab+cd$.
57. $ab-cd$. 58. $3m-n$. 59. $m-3n$. 60. $3m-4n$.
61. $4m-3n$. 62. $x+3$. 63. $x+7$. 64. $7x+1$.
65. $x-4$. 66. $4x-1$. 67. $2x-5$. 68. $5x-2$.
69. $3x+8$. 70. $8x-3$. 71. $\frac{1}{2}x+1$. 72. $x+\frac{1}{2}$.
73. $x-\frac{1}{4}$. 74. $\frac{1}{4}x-1$.

Find the square root of the following :

75. $g^2+2gh+h^2$. 76. $g^4+2g^2h^2+h^4$.
77. $g^6+2g^3h^3+h^6$. 78. $g^8+2g^4h^4+h^8$.
79. $k^2-2kl+l^2$. 80. $k^6-2k^3l^3+l^6$.
81. $\frac{1}{4}k^2+\frac{1}{2}kl+\frac{1}{4}l^2$. 82. $\frac{1}{4}k^2-\frac{1}{2}kl+\frac{1}{4}l^2$.
83. $9a^2+6ab+b^2$. 84. $a^2+6ab+9b^2$.
85. $4x^2+20xy+25y^2$. 86. $25x^2-20xy+4y^2$.
87. $9p^2+42pq+49q^2$. 88. $49p^2-42pq+9q^2$.
89. $4m^2+36mn+81n^2$. 90. $81m^2-36mn+4n^2$.
91. $25x^2+60x+36$. 92. $36x^2-60x+25$.
93. $81x^2+180x+100$. 94. $100x^2-180x+81$.

$2^3 = 2 \times 2 \times 2 = 8$, and $\sqrt[3]{8} = 2$.
$3^3 = 3 \times 3 \times 3 = 27$, and $\sqrt[3]{27} = 3$.
$1^3 = 1 \times 1 \times 1 = 1$, and $\sqrt[3]{1} = 1$.
$x^3 = x \times x \times x$, and $\sqrt[3]{x^3} = x$.

Number.		Cube.
0	..	0
1	..	1
2	..	8
3	..	27
4	..	64
5	..	125
6	..	216
7	..	343
8	..	512
9	..	729
10	..	1000

From the table on this page you can find the cube of any number up to 10, and approximately the cube root of any number up to 1,000.

When we cube numbers with indices we treble the indices. Thus $(x^n)^3 = x^{3n}$. When we extract the cube root we take a third of the index. Thus $\sqrt[3]{x^{12}} = x^4$.

Keeping the brackets as they are, write out the cubes of the following :

1. 4. 2. 9. 3. m. 4. m^4.
5. m^{10}. 6. 1^2. 7. abc. 8. $3xy^2z^3$.
9. $(x + y)$. 10. $4(a-7)^3$. 11. $\left(\dfrac{m+n}{m}\right)^2$.
12. $2(a^2 + b^2)^3$. 13. $\frac{1}{2}$.
14. $\frac{2}{3}(p - q)$. 15. $\frac{7}{9}(p^2 - p + 1)^4$.

Extract the cube root of each of the following :

16. 512. 17. $\frac{125}{343}$. 18. $\frac{1}{8}(a + b)^3$.
19. $(n - 1)^6$. 20. $\dfrac{1}{27}\left(\dfrac{m+n}{p}\right)^9$.
21. $(p - q)^{3x}$. 22. $\frac{27}{125}(2p - q)^{6x}$.

Find to the nearest whole number the cubic root of :

23. 200. 24. 550. 25. 120. 26. 800.
27. Find by multiplication the value of: (i) $(a + b)^3$, (ii) $(x + y)^3$.
28. Now write out, without multiplying, the value of $(m + n)^3$.
29. Find the cube root of $p^3 + 3p^2q + 3pq^2 + q^3$.
30. Find by multiplication the value of: (i) $(a - b)^3$, (ii) $(x - y)^3$.

EXERCISE 40.—SQUARES AND CUBES OF NEGATIVE NUMBERS

$$(-2)^2 = (-2) \times (-2) = 4.$$
$$(-3)^2 = (-3) \times (-3) = 9.$$
$$(-x)^2 = (-x)(-x) \quad = x^2.$$

The square of every number, even if it is a negative number, is always positive.

4 has really two square roots, 2, and -2; and x^2 has really two square roots, x and $-x$. But we generally ignore the negative root.

$$(-x)^3 = (-x)(-x)(-x) = -x^3.$$

A negative number cubed is always negative.

Square the following without disturbing the brackets:

1. 9. 2. -9. 3. $-(a+b)$.

4. $-(x-y)^3$. 5. $-\left(\frac{x+y}{2}\right)^4$. 6. $-\frac{2}{3}(m-n)^x$.

Cube the following without disturbing the brackets:

7. -6. 8. $-\frac{3}{4}$. 9. $-(x-y)^2$.

10. $-\left(\frac{a+b}{a-b}\right)^3$. 11. $-\frac{2}{5}\left(\frac{p+q}{3}\right)^4$. 12. $-(n-1)^{2x}$.

Find the value of the following expressions when $x = -2$:

13. $x^3 - x^2 + x - 1$. 14. $x^3 + x^2 - x + 1$.
15. $3x^3 + 4x^2 + 2x - 3$. 16. $5x^3 - 3x^2 + 4x - 1$.
17. $x^5 + 2x^4 - 3x^3 + 5x^2 - 3x + 1$.
18. $2x^6 - 3x^5 + x^4 - x^3 + 2x^2 - 4x + 2$.

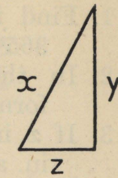
Find the value of the following when $x = -3$.

19. $x^3 + x^2 - x - 1$. 20. $x^3 - x^2 + x + 1$.
21. $2x^3 - 3x^2 + 2x - 2$. 22. $2x^3 - x^2 - 2x - 3$.
23. $3x^3 - 2x^2 - 3x + 4$. 24. $4x^3 + 2x^2 - x + 3$.

Find the value of the following when $x = -1$.

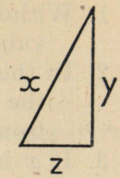
25. $x^5 + x^4 - x^3 + x^2 - x + 1$.
26. $x^5 - x^4 + x^3 - x^2 + x - 1$.
27. $2x^6 - 3x^5 + 4x^4 - x^3 + 2x^2 - 3x + 4$.
28. $5x^7 - 3x^6 + 2x^5 - 4x^4 + x^3 - 3x^2 + 2x - 1$.

EASY TEST 7



1. Without squaring the numbers, find the value of $58^2 - 56^2$.
2. In this right-angled triangle, which side is the hypotenuse? Give the formula from which its value may be found.
3. If x in the accompanying triangle is 5 in. and z is 3 in., how long is y?
4. Simplify: (i) $y^2 \times y^5$, (ii) $p^a \times p^b$, (iii) $(x + y)(x + y)^2$.
5. What is the square of $(b - c)^4$?
6. Simplify: (i) $\frac{a^5}{a}$, (ii) $\frac{(c + d)^7}{(c + d)^5}$, (iii) $\frac{(d - f)^4}{(d - f)^4}$.
7. Simplify $(x + y)^3 \div (x + y)^7$.
8. Find the value of these products:
   (i) $(b + c)(b - c)$.
   (ii) $(3x + y)(3x - y)$.
   (iii) $(4n + 1)(4n - 1)$.
9. Simplify $(p + q)(p - q)(p^2 + q^2)$.
10. Factorize the following:
    (i) $d^2 - f^2$.
    (ii) $4a^2 - 9b^2$.
    (iii) $25p^2 - 16q^2$.
11. Simplify:
    (i) $(2a + b)^2 - 4ab$.
    (ii) $(x - 3y)^2 + 6xy$.
    (iii) $(a + 3)^2 - (a + 2)^2$.
12. What is the square of $4a^2b^3$?
13. What is the square root of $\frac{9x^2}{16y^4}$?
14. What is the cube of $-(p - q)$? (Keep the brackets.)
15. If $a = 1, b = 3, c = 5$, and $d = 0$, find the value of—
    $$\sqrt{3ab} + \sqrt{5ac} + \sqrt{15bc} + \sqrt{4cd}.$$
16. Which is the larger, and by how much: (i) the square of $3a^3$, or (ii) the cube of $2a^2$?
17. Find the value of the following expressions if $x = -2$.
    (i) $5x^3 - 4x^2 + 3x - 2$, (ii) $2x^3 + 3x^2 - 4x + 5$.

1. Find in the simplest way the value of $365^2 - 362^2$.
2. In this right-angled triangle give the formula for finding the value of y.
3. If x in the accompanying triangle is 13 in. and y 12 in., how long is z?



4. Simplify:
   (i) $a \times a^5 \times a^7$.
   (ii) $(q + r)^3(q+r)^5$.
   (iii) $\left(\frac{x}{y}\right)^2\left(\frac{x}{y}\right)^5$.
5. What is the square of $(a + b)^n$?
6. Simplify: (i) $\frac{b^2}{b^5}$, (ii) $\frac{(x - y)^4}{(x - y)}$, (iii) $\frac{(p - r)^n}{(p - r)^n}$.
7. Simplify $(a + b - c) \div (a + b - c)^4$.
8. Find the value of these products:
   (i) $(3a + 4b)(3a - 4b)$.
   (ii) $(5ab - 3)(5ab + 3)$.
   (iii) $(\frac{1}{2}x + \frac{1}{4}y)(\frac{1}{2}x - \frac{1}{4}y)$.
9. Simplify $(2x - y)(2x + y)(4x^2 + y^2)$.
10. Factorize the following:
    (i) $a^2b^2 - 1$.
    (ii) $9x^2y^2 - 4y^2z^2$.
    (iii) $\frac{1}{9}m^2 - \frac{1}{4}n^2$.
11. Simplify:
    (i) $(2x + 3)^2 + (3x + 2)^2$.
    (ii) $9a^2 + 16b^2 - (3a - 4b)^2$.
    (iii) $(2p + 3q)^2 - (2p + q)(2p - q)$.
12. What is the square of $-5a^3b^2c$?
13. What is the square root of $\frac{4(a + b)^4}{25y^6}$?
14. What is the cube of $-(x - 2y)$? (Keep the brackets.)
15. If $x = 2, y = 3, z = 4$, and $p = 0$, find the value of—

$$\sqrt{12yz} - \sqrt{6xy} + \sqrt{5pz} - \sqrt{2xz}.$$

16. If $x = -3$, what is the value of $3x^3 + 4x^2 + 5x - 1$?

EXERCISE 41.—DIVISION

Compare these two division sums :

$$\begin{array}{r} 3\overline{)9 \quad 3 \quad 6} \\ \hline 3 \quad 1 \quad 2 \end{array} \qquad\qquad \begin{array}{r} 3\overline{)9x^2 + 3x + 6} \\ \hline 3x^2 + x + 2 \end{array}$$

These are the same when $x = 10$.

Divide :

1. $x^3 + 2x^2 + x$ by x.
2. $2x^3 - 6x^2 + 4x$ by $2x$.
3. $a^2 + ab + ac$ by a.
4. $2a^2b - ab^2 - abc$ by ab.
5. $a^4 + a^3b - a^2b^2$ by a^2.
6. $3ab - 3ac + 3ad$ by $3a$.
7. $2m^3 - 4m^2n + 2mn^2$ by $2m$.
8. $5x^4y - 5x^3y^2 - 5x^2y^3$ by $5x^2y$.

Long division in algebra is like long division in arithmetic, e.g. :

$$\frac{2a^2 + 5ab + 2b^2}{a + 2b} \qquad\qquad \begin{array}{r} 2a + b \\ a + 2b\overline{)2a^2 + 5ab + 2b^2} \\ 2a^2 + 4ab \\ \hline ab + 2b^2 \\ ab + 2b^2 \\ \hline \end{array}$$

Divide :

9. $x^2 + 3x + 2$ by $x + 1$.
10. $2a^2 - ab - b^2$ by $a - b$.
11. $2a^2 + ab - b^2$ by $a + b$.
12. $m^3 + m^2n - mn^2 - n^3$ by $m + n$.
13. $2p^2 + 3pq - 2q^2$ by $2p - q$.
14. $a^3b - 2a^2b^2 + ab^3$ by $ab - b^2$.
15. $6a^2 + ab - 2b^2$ by $3a + 2b$.
16. $x^3y - 2x^2y^2 + xy^3$ by $x^2 - xy$.

As in arithmetic, there is sometimes a remainder, which may be written as a fraction, e.g. $(a^2 + 3ab + b^2) \div (a + b) = a + 2b$, with a remainder $- b^2$. The quotient therefore is— $\quad a + 2b - \dfrac{b^2}{a + b}$.

17. $(a^2 + 2ab + 2b^2) \div (a + b)$.
18. $(x^2 - 3xy - 3y^2) \div (x - y)$.
19. $(x^2 + x - 8) \div (x - 3)$.

To factorize 28 is to write it as 4×7 or $2 \times 2 \times 7$.

To factorize $a^2 - ab$ is to write it as $a(a + b)$. ? —

To factorize $-2xy - 4y^2$ is to write it as $-2y(x + 2y)$.

Factorize the following :

1. $ab - b^2$.
2. $a^2b + a^2c - a^2d$.
3. $3a^2b + 6ab^2 - 9b^3$.
4. $-m^3n^3 - m^2n^2 - mn$.
5. $4x^2 + 8xy - 2xz$.
6. $-5x^2y + 15xy^2 - 10y^3$.

Study the example in Exercise 19 and then look at this mode of factorization :

$$\begin{aligned} ac + ad + bc + bd &= a(c + d) + b(c + d) \\ &= (a + b)(c + d). \end{aligned}$$

Employ this method to factorize :

7. $mp + mr + np + nr$.
8. $ac - ad + bc - bd$.
9. $ac + ad - bc - bd$.
10. $ac - ad - bc + bd$.
11. $2pr + 4ps + qr + 2qs$.
12. $2pr - 4ps + qr - 2qs$.
13. $2pr - 4ps - qr + 2qs$.
14. $2pr + 4ps - qr - 2qs$.

Look at this multiplication :

$$\begin{array}{l} x + 5 \\ \underline{x + 3} \\ x^2 + 5x \\ \underline{\qquad\ 3x + 15} \\ \underline{x^2 + 8x + 15} \end{array}$$

To factorize $x^2 + 8x + 15$ we must think of two numbers whose sum is 8 and whose product is 15. Those two numbers are 3 and 5. Therefore the factors are $(x + 3)$, $(x + 5)$ or, if you like, $(x + 5)$, $(x + 3)$.

Now factorize the following :

15. $x^2 + 5x + 6$.
16. $x^2 + 6x + 8$.
17. $y^2 + 4y + 3$.
18. $y^2 + 7y + 12$.
19. $a^2 + 6a + 5$.
20. $a^2 + 7a + 10$.
21. $a^2 + 10a + 21$.
22. $m^2 + 9m + 20$.
23. $m^2 + 10m + 24$.
24. $m^2 + 11m + 30$.
25. $p^2 + 11p + 24$.
26. $r^2 + 9r + 14$.
27. $b^2 + 11b + 28$.
28. $c^2 + 10c + 9$.
29. $t^2 + 9t + 18$.
30. $d^2 + 13d + 40$.

EXERCISE 43.—FACTORS III

Look at this multiplication :

$$
\begin{array}{l}
x - 4 \\
x - 3 \\
\hline
x^2 - 4x \\
\quad\;\; - 3x + 12 \\
\hline
x^2 - 7x + 12
\end{array}
$$

The factors of $x^2 - 7x + 12$ are $(x - 3)$, $(x - 4)$. To find the factors, look for two numbers whose sum is 7 and whose product is 12.

Factorize the following :

1. $x^2 - 7x + 10$.
2. $x^2 - 8x + 15$.
3. $x^2 - 3x + 2$.
4. $x^2 - 11x + 28$.
5. $a^2 - 8a + 12$.
6. $a^2 - 12a + 35$.
7. $m^2 - 9m + 18$.
8. $m^2 - 14m + 48$.

Look at these two multiplication sums :

$$
\begin{array}{l}
x - 4 \\
x + 3 \\
\hline
x^2 - 4x \\
\qquad\; 3x - 12 \\
\hline
x^2 - \;\; x - 12
\end{array}
\qquad\qquad
\begin{array}{l}
x + 4 \\
x - 3 \\
\hline
x^2 + 4x \\
\quad\;\; - 3x - 12 \\
\hline
x^2 + \;\; x - 12
\end{array}
$$

When the third term of the product is negative, one of the factors has a plus and the other a minus. To find the factors think of two numbers whose difference is the coefficient of the middle term (1 in this case) and whose product is the last term. When the middle term is positive, the larger of the two numbers is positive; when the middle term is negative, the larger of the two numbers is negative.

Factorize the following :

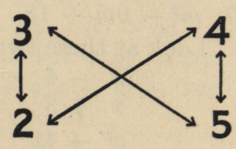
9. $x^2 - 2x - 8$.
10. $x^2 + 3x - 18$.
11. $x^2 - x - 2$.
12. $x^2 - 3x - 28$.
13. $y^2 + 4y - 12$.
14. $y^2 + y - 30$.
15. $a^2 - 2a - 15$.
16. $a^2 + 3a - 54$.
17. $b^2 - b - 20$.
18. $b^2 + 5b - 14$.
19. $m^2 - 3m - 4$.
20. $m^2 - 3m - 40$.

$$\begin{array}{rrr} 3a & +\,4b & \\ 2a & +\,5b & \\ \hline 6a^2 & +\,8ab & \\ & 15ab & +\,20b^2 \\ \hline 6a^2 & +\,23ab & +\,20b^2 \\ \hline \end{array} \qquad \begin{array}{rrr} 3a & -\,4b & \\ 2a & -\,5b & \\ \hline 6a^2 & -\,8ab & \\ & -\,15ab & +\,20b^2 \\ \hline 6a^2 & -\,23ab & +\,20b^2 \\ \hline \end{array}$$

$$\begin{array}{rrr} 3a & +\,4b & \\ 2a & -\,5b & \\ \hline 6a^2 & +\,8ab & \\ & -\,15ab & -\,20b^2 \\ \hline 6a^2 & -\,7ab & -\,20b^2 \\ \hline \end{array} \qquad \begin{array}{rrr} 3a & -\,4b & \\ 2a & +\,5b & \\ \hline 6a^2 & -\,8ab & \\ & 15ab & -\,20b^2 \\ \hline 6a^2 & +\,7ab & -\,20b^2 \\ \hline \end{array}$$

Work these four sums for yourselves. Then put the four products down like this :

(*a*) $6a^2 + 23ab + 20b^2$.
(*b*) $6a^2 - 23ab + 20b^2$.
(*c*) $6a^2 - 7ab - 20b^2$.
(*d*) $6a^2 + 7ab - 20b^2$.



How can you find the factors by looking at the products ?

If you will arrange the four coefficients of the factors as above, you will see that the first and last terms of the product are obtained by multiplying vertically, and the middle term by multiplying obliquely.

Finish these sentences :

1. When the last term in the product is positive, the signs in the factors are . . .
2. When the last term in the product is negative, the signs in the factors are . . .
3. A difference of signs in the factors makes a difference in the coefficient of one term only of the product, and that is the . . . term.

Now answer these questions :

4. What other factors besides $3a$ and $2a$ *might* have made the product $6a^2$?
5. What other factors besides $4b$ and $5b$ *might* have made the product $20b^2$?

6. The middle term is either the . . . or the . . . of the two oblique multiplications. What are the missing words ?

Factorize the following :

7. $6x^2 + 17x + 12$.
8. $6x^2 - x - 12$.
9. $6x^2 + x - 12$.
10. $6x^2 - 17x + 12$.
11. $15a^2 + ab - 2b^2$.
12. $15a^2 - 11ab + 2b^2$.
13. $15a^2 - ab - 2b^2$.
14. $4x^2 - 11xy + 6y^2$.
15. $4x^2 - 5xy - 6y^2$.
16. $4x^2 + 5xy - 6y^2$.
17. $6m^2 + 23mn + 7n^2$.
18. $6m^2 - 19mn - 7n^2$.
19. $6m^2 - 23mn + 7n^2$.
20. $6m^2 - 19mn - 7n^2$.
21. $3a^2 + 10ab + 8b^2$.
22. $3a^2 - 10ab + 8b^2$.
23. $3a^2 + 2ab - 8b^2$.
24. $3a^2 - 2ab - 8b^2$.
25. $6x^2 + 13x + 5$.
26. $6x^2 + 7x - 5$.
27. $6x^2 - 13x + 5$.
28. $6x^2 - 7x - 5$.
29. $4c^2 - 15cd + 9d^2$.
30. $4c^2 + 9cd - 9d^2$.
31. $4c^2 - 9cd - 9d^2$.
32. $4c^2 + 15cd + 9d^2$.
33. $10e^2 - 11ef - 6f^2$.
34. $10e^2 + 19ef + 6f^2$.
35. $10e^2 + 11ef - 6f^2$.
36. $10e^2 - 19ef + 6f^2$.
37. $6y^2 - 5y + 1$.
38. $6y^2 + 5y + 1$.
39. $6y^2 + y - 1$.
40. $6y^2 - y - 1$.
41. $3m^2 + 8mn + 4n^2$.
42. $3m^2 + 4mn - 4n^2$.
43. $3m^2 - 8mn + 4n^2$.
44. $3m^2 - 4mn - 4n^2$.
45. $4p^2 - 17pq + 4q^2$.
46. $4p^2 - 15pq - 4q^2$.
47. $4p^2 + 17pq + 4q^2$.
48. $4p^2 + 15pq - 4q^2$.
49. $3x^2 + 11x + 6$.
50. $3x^2 + 7x - 6$.
51. $3x^2 - 7x - 6$.
52. $3x^2 - 11x + 6$.
53. $4b^2 + 12bc + 5c^2$.
54. $4b^2 - 12bc + 5c^2$.
55. $4b^2 - 8bc - 5c^2$.
56. $4b^2 + 8bc - 5c^2$.
57. $20w^2 - 3wx - 2x^2$.
58. $20w^2 + 13wx + 2x^2$.
59. $20w^2 + 3wx - 2x^2$.
60. $20w^2 - 13wx + 2x^2$.
61. $12y^2 - 29y + 15$.
62. $12y^2 - 11y - 15$.
63. $12y^2 + 11y - 15$.
64. $12y^2 + 29y + 15$.
65. $6p^2 + 13pq - 28q^2$.
66. $6p^2 - 13pq - 28q^2$.
67. $6p^2 - 29pq + 28q^2$.
68. $6p^2 + 29pq + 28q^2$.
69. $35r^2 + 11rs - 6s^2$.
70. $35r^2 + 31rs + 6s^2$.
71. $35r^2 - 31rs + 6s^2$.
72. $35r^2 - 11rs - 6s^2$.

EXERCISE 45.—SIMPLIFICATION OF FRACTIONS

Just as $\frac{2}{3} \times \frac{5}{7} = \frac{2 \times 5}{3 \times 7}$, so $\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}$.

And as $\frac{2}{3} \div \frac{5}{7} = \frac{2}{3} \times \frac{7}{5}$, so $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c} = \frac{ad}{bc}$.

We also cancel in algebra as in arithmetic.

Just as $\frac{2 \times 3 \times 5}{4 \times 6 \times 5} = \frac{2 \times 3}{2 \times 2 \times 2 \times 3} = \frac{1}{2 \times 2} = \frac{1}{4}$,

so $\frac{a^2x - abx}{abx^2 - b^2x^2} = \frac{ax(a-b)}{bx^2(a-b)} = \frac{ax}{bx^2} = \frac{a}{bx}$.

Simplify the following :

1. $\frac{a}{b} \times 2$.
2. $\frac{x}{4} \times a$.
3. $\frac{a}{b} \div c$.
4. $\frac{a}{b} \times \frac{b}{c}$.
5. $\frac{abc}{ab^2d}$.
6. $\frac{4m^2n}{2mn}$.
7. $\frac{x+y}{x-y} \times \frac{a+b}{x+y}$.
8. $\frac{x^2-y^2}{x+y}$.
9. $\frac{x^2-y^2}{x-y}$.
10. $\frac{a^3+a^2b+ab^2}{a^2b+ab^2+b^3}$.
11. $\frac{x^2+3x+2}{x^2+4x+3}$.
12. $\frac{a^2+3ab+2b^2}{a^2-ab-2b^2}$.
13. $\frac{ac+ad+bc+bd}{ae+af+be+bf}$.
14. $\frac{6x^2-17x+12}{2x^2-7x+6}$.
15. $\frac{a(x+y)-b(x+y)}{a-b}$.
16. $\frac{x^2-y^2}{(x+y)(x-y)}$.
17. $\frac{x^2-y^2}{x^2+2xy+y^2}$.
18. $\frac{x^2-y^2}{x^2-2xy+y^2}$.
19. $\frac{2a^2+a-3}{8a^2+4a-12}$.
20. $\frac{5m+10n}{5} - \frac{4m-12n}{4}$.
21. $\frac{18p-4q}{2} - \frac{6p+9q}{3}$.
22. $\frac{6x^2+9x-3}{3} - \frac{8x^2+16x+12}{4}$.

EASY TEST 8

1. Divide $9x^3 - 12x^2y + 3xy^2$ by $3x$.
2. Divide $x^2 + 5x + 6$ by $x + 2$.
3. If $x - 1$ is one of the factors of $3x^2 + x - 4$, what is the other factor ?
4. Factorize :

   (i) $x^2 + xy$.
   (ii) $2a^3 + 6a^2b - 2ab^2$.
   (iii) $-9m^3n + 6m^2n^2 - 3mn^3$.
5. Resolve into factors :

   (i) $a^2 - a - 12$.
   (ii) $3b^2 + 7bc + 2c^2$.
   (iii) $2x^2 + 5x - 12$.
6. Simplify :

   (i) $\left(\frac{x}{y}\right)\left(\frac{c}{d}\right)$. (ii) $\frac{3(a+b)}{6(a+b)^2}$.

   (iii) $\frac{a(x-y)(x-2y)}{ab(x-y)(x+2y)}$.
7. Simplify :

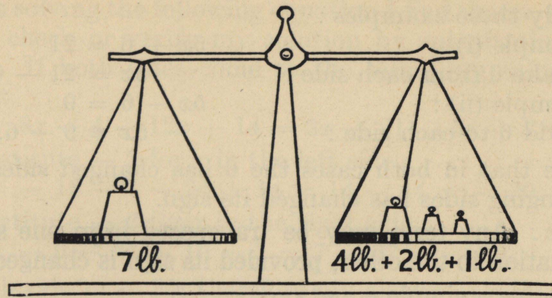
   (i) $\frac{ab}{c+d} \div \frac{a^2b}{(c+d)^2}$. (ii) $\frac{(p-q)(p+q)}{p^2 - 2pq + q^2}$.

   (iii) $\frac{mx + my}{nx^2 + 2nxy + ny^2}$.
8. Remove brackets and simplify :

   $$3(x^2 + 2x - 2) - 2(x^2 - 5x - 3).$$
9. When $x = -2$, what is the value of $x^3 + x^2 - x + 4$?
10. If $a = 1$, $b = 3$, $c = 5$, and $d = 0$, find the value of $ab^2 - ad^2 + bc^2 - \sqrt{cd} + \sqrt{5ac}$.
11. What must be added to $a + b$ to make $c - d$?
12. If p pounds are worth q francs, how many francs can I get for a pound ? How many for r pounds ?
13. By what must I divide $a + b$ to make it x ?
14. Simplify $\frac{9x^2 - 12xy + 3y^2}{3} - \frac{10x^2 - 15xy + 5y^2}{5}$.

1. Divide $12x^3y^2 - 16x^2y^3 + 8xy^4$ by $4xy^2$.
2. Divide $6x^2 + 2xy - 20y^2$ by $2x + 4y$.
3. If $3a - 2b$ is one of the factors of $12a^2 + ab - 6b^2$, what is the other factor ?
4. Factorize :
   (i) $a^3b - a^2b^2 + ab^3$.
   (ii) $ad + af + bd + bf$.
   (iii) $6px - 4py + 9qx - 6qy$.
5. Resolve into factors :
   (i) $m^2 - 4m + 3$.
   (ii) $x^2 - 10x + 21$.
   (iii) $8x^2 - 14xy - 15y^2$.
6. Simplify :
   (i) $\left(\dfrac{a^2b}{b^2}\right)\left(\dfrac{ab}{a^2b^2}\right)$.
   (ii) $\dfrac{a^2 + 2ab + b^2}{a^2 - b^2}$.
   (iii) $\dfrac{(x^4 - y^4)(x^2 - 2xy + y^2)}{(x - y)(x^2 - y^2)(x^2 + y^2)}$.
7. Simplify :
   (i) $\dfrac{cd(c + d)}{(d - f)} \div \dfrac{(c + d)}{df(d - f)}$.
   (ii) $\dfrac{(t + v)(t - v)tv}{t^2v(t^2 + 2tv + v^2)}$. (iii) $\dfrac{x^2 + 7x + 12}{x^2 + 8x + 15}$.
8. Remove the brackets and simplify :
   $2x(x + y) - 3y(x - y) - x(2x - y)$.
9. When $a = -3$, what is the value of—
   $2a^3 + 3a^2 - 5a + 6$?
10. If $x = 2$, $y = 4$, $z = 6$, and $w = 0$, find the value of—
    $$\frac{x + y}{z} - \frac{z - x}{y} + \frac{wx}{z} + \sqrt{y + 2z} - \frac{w - y}{x}.$$
11. What is the nth term in the series : 7, 14, 21, 28, etc. ?
12. A man earns p shillings a week, and his son q shillings a week. How much will they together earn in n weeks ?

EXERCISE 46.—EQUALITY



$$7 = 4 + 2 + 1.$$

$\times$ by 2 : $\quad 14 = 8 + 4 + 2.$

$\div$ by 2 : $\quad 3\frac{1}{2} = 2 + 1 + \frac{1}{2}.$

$+ 2$: $\quad 7 + 2 = 4 + 2 + 1 + 2.$

$- 2$: $\quad 7 - 2 = 4 + 2 + 1 - 2.$

All the above are statements of equality. Each has two sides of the same value separated by the sign $=$. So long as I treat both sides alike they still remain equal.

$x + 4 = 7$ is called an *equation*. It is not true for every value of x, but only for one particular value of x. In this instance the assertion is true only if $x = 3$.

To solve an equation is to find the value of x which will make the statement true.

Example (i) :

$$x + 9 = 21.$$

Take 9 from each side: $\quad x = 21-9.$

$$x = 12.$$

Example (ii) :

$$3x = 15.$$

Divide both sides by 3 : $\quad x = 5.$

Solve the following :

1. $x + 4 = 6.$ 2. $x - 7 = 10.$ 3. $x - 15 = 3.$
4. $14 + x = 18.$ 5. $7 + x = 7.$ 6. $x - 4 = 26.$
7. $8x = 40.$ 8. $\frac{x}{8} = 3.$ 9. $7x = 84.$ 10. $\frac{x}{5} = 20.$
11. $\frac{x}{15} = 5.$ 12. $19x = 57.$ 13. $\frac{x}{21} = 4.$ 14. $4x = 2.$

EXERCISE 47.—SIMPLE EQUATIONS

Study these examples :

Example (i) : $5x + 6 = 21$

Take 6 from each side : $5x = 21 - 6$

Example (ii) : $5x - 6 = 9$

Add 6 to each side : $5x = 9 + 6.$

Note that in both cases the 6 has changed sides, and in changing sides has changed its sign.

Rule: **Any term may be transposed from one side of an equation to the other, provided its sign is changed.**

To solve a simple equation :

1st Step : Simplify each side.

2nd Step : Bring all the terms involving x to one side of the equation and all the terms not involving x to the other.

3rd Step : Simplify each side.

4th Step : Find the value of x by division.

E.g. Solve : $3x - 6 + 8x + 4 = 5x - 2x + 5 - 3$

1st step : $11x - 2 = 3x + 2$

2nd step : $11x - 3x = 2 + 2$

3rd step : $8x = 4$

4th step : $x = \frac{4}{8} = \frac{1}{2}.$

Solve the following equations by this method :

1. $4x + 7 - 5 = 3x - 2 + 6.$
2. $6x + 5 - 2x = 4x - 3x + 8.$
3. $7 + 3x - 4 + 6x = 8x - 2x + 12.$
4. $8x - 9 - x + 3 = x + 12 + 3x - 3.$
5. $4x - 3 + 8x = 5x - 2 - 3x + 4.$
6. $7x - 5 - x - 2 = 8x - 9 - 5x + 4.$
7. $9x + 4 - 6x - 7 = 2x - 8 - 4x.$
8. $x + 5 + 9x + 3 = 2x - 6 + 5x + 2.$
9. $3x - 28 - 4x + 12 = x + 32 - 6x - 8.$
10. $7x - 8 - 2x + 1 = 3x + 7 - 13x - 2.$
11. $16x - 9 + 14x - 6 = 7x - 13 + 5x - 8.$
12. $23x + 7 - 5x - 1 = x - 8 - 7x + 4.$
13. $5x - 3 + 8x + 1 = 2x - 5 + 9x - 2.$

EXERCISE 48.—VERIFICATION

If in solving the following equation I find that $x = -2$, I can check or verify my solution by substituting -2 for x. If both sides come to the same, my solution is correct.

$$8x - 4 + 12x + 14 = 3x - 21 + 5x + 7.$$

Substitute -2 for x in the left side :

$$-16 - 4 - 24 + 14 = 14 - 44 = -30.$$

Substitute -2 for x in the right side :

$$-6 - 21 - 10 + 7 = 7 - 37 = -30.$$

Each side $= -30$, therefore my answer is right.

Some of the solutions of the following equations are right and some are wrong. Find out by substitution which are right and which are wrong.

1. $7x - 8 = 4x + 7$. Is $x = 5$ right ?
2. $4x - 13 = 8 - 3x$. Is $x = 3$ right ?
3. $6 - 2x = 4x - 18$. Is $x = 2$ right ?
4. $20x + 7 = 5x - 8$. Is $x = -1$ right ?
5. $2x + 6 + 6x = 3x - 9$. Is $x = 3$ right ?
6. $9x - 6 + 3x = 2 + 2x - 3$. Is $x = \frac{1}{2}$ right ?
7. $4x + 6 + 8x - 2 = 2x - 7 + 5x - 9$. Is $x = -4$ right ?
8. $5x - 4 + 2x - 7 = 3x + 5 - 4x - 3$. Is $x = -2$ right ?
9. $6x + 4 - 3x - 5 = 9x - 6 + 6x + 2$. Is $x = \frac{1}{3}$ right ?
10. $10x + 4 + 3x - 3 = 2x - 2 - 5x - 9$. Is $x = -\frac{3}{4}$ right ?
11. $3x + 7 - 4x + 3 = 6 + 8x - 5$. Is $x = 2$ right ?
12. $9 + 10x - 2 = 2x - 4 + 3x + 1$. Is $x = -2$ right ?
13. $3x - 4 - x - 6 = 2x + 8 - 6x$. Is $x = 3$ right ?
14. $9x + 5 - 2x + 3 = 4 - 4x - 7$. Is $x = -1$ right?
15. $\dfrac{5x}{17} = \dfrac{10}{102}$. Which is right, $x = \frac{1}{5}$ — $x = \frac{1}{3}$?
16. $\dfrac{3x}{25} = \dfrac{4}{37}$. Which is right, $x = \dfrac{100}{111}$, or $x = \frac{1}{2}$?

EXERCISE 49.—CHANGING SIDES

Look at this equation : $18 = 3x$.

This is the same as $3x = 18$.

Suppose I follow the rule and change the signs, so that the equation becomes $-3x = -18$.

By multiplying both sides by -1 I get—

$$3x = 18.$$

It is clear, therefore, that I can, if I like, change the two sides over bodily without changing the signs.

It is sometimes convenient to multiply both sides by -1, e.g. : $-4x = 12$.

Multiply by -1 : $4x = -12$.

$\therefore \quad x = -3$.

Solve the following equations, and check your answers by substitution :

1. $14 + 6 - 2 = 5x + 3x - 2x$.
2. $12 + 5 - 3 = 9x - 6x - x$.
3. $2 - 8 + 12 - 4 = 7x - 3x + 2x - 4x$.
4. $5 - 7 + 3 - 9 = 2x - 4x - 3x + 7x$.
5. $5x - 10 = 9x - 2$.
6. $2x + 4 - 5x = 3x + 7$.
7. $3x - 6 + 2x + 4 = 5x + 3 + x$.
8. $2x - 10 - 4x + 1 = 4 + 5x + 8$.
9. $6 - x + 2x - 4 = 6x + 5 + 4x + 3$.
10. $4x + 3 - 9x - 6 = 8x - 4 - 5x + 7$.
11. $4x - 14 - 3x - 9 = 2x + 7 - 8x - 2$.
12. $5x + 10 - 2x + 8 = x - 4 - 3x + 2$.
13. $7x - 12 + 6x + 6 = 9x + 6 - 3x + 2$.
14. $13x - 10 - 6x + 3 = 8x + 5 - 7x - 9$.
15. $12x + 10 + 8x - 8 = 3x + 2 + 2x - 15$.
16. $11x - 9 + 4x - 10 = 6x - 8 - 3x - 7$.
17. $5x - 7 + 8x + 5 = 6x + 15 - 9x - 13$.
18. $3x - 10 + 9x - 3 = 2x - 4 - 6x - 1$.
19. $6x - 4 - 2x + 2 = 9 - 10x - 5 - 4x$.
20. $4x - 12 - 2x + 3 = 3x + 13 - 6x - 2$.
21. $9x - 7 + 6x + 8 - 8x = 5x + 4 - 10x - 9$.
22. $9x + 4 - 3x + 1 = 4 + x - 7 - 10x + 3$.
23. $8x + 5 + 7x - 6 - 3x = 3 + 2x - 2 - 12x - 8$.

EXERCISE 50.—MENSURATION FORMULÆ I

Look at this statement : The number of square units in the area of a rectangle is equal to the number of linear units in the length multiplied by the number of linear units in the breadth.

Put more briefly : The area of a rectangle is found by multiplying the length by the breadth.

Put more briefly still : $A = lb$.

Consider this simple problem : If the area of a rectangle is 24 sq. in. and its length is 8 in., what is its breadth ?

Substitute for the letters in the formula : $24 = 8x$.

This is now an equation. Change sides : $8x = 24$.

Divide by 8 : $x = 3$.

The breadth is 3 in.

Nearly all the formulæ in simple mensuration are of this kind. There is only one term on the left and only one on the right. They involve multiplication and division, but not addition and subtraction. And when all the numbers are known except one, the formula becomes an equation.

Look at this formula : $pr = st$.

This is a sentence of which pr is the subject and " = " is the verb.

To make p the subject, divide by r : $p = \frac{st}{r}$

To make s the subject, change sides and divide by t : $s = \frac{pr}{t}$

1. Change the formula $A = lb$ so that b becomes the subject. What is the value of b when $A = 224$ sq. ft., and $l = 16$ ft. ?
2. Change the formula $c = \pi d$ so that d becomes the subject. Find the value of d when c is 9·4248 in. ($\pi = 3{\cdot}1416$.)
3. Change the formula $c = 2\pi r$ so that r becomes the subject. Find the value of r when $c = 15{\cdot}708$ cm.
4. Change the formula $A = \pi r^2$ so that r^2 is the subject. Find the value of r^2 when $A = 12{\cdot}5664$ sq. in.

Look at the statement: $\frac{2}{4} = \frac{3}{6}$

Multiply diagonally: $2 \times 6 = 4 \times 3$

Turn the fractions upside down: $\frac{4}{2} = \frac{6}{3}$

Is this true of all equivalent fractions, such as $\frac{1}{3} = \frac{2}{6}$?

It is always true. Suppose $\frac{a}{b} = \frac{c}{d}$ . . . . (i)

Multiply by bd: $\frac{abd}{b} = \frac{cbd}{d}$

$\therefore \quad ad = bc$ . . . . (ii)

Change sides: $bc = ad$

Divide by ac: $\frac{bc}{ac} = \frac{ad}{ac}$

$\therefore \quad \frac{b}{a} = \frac{d}{c}$ . . . . (iii)

I can find the value of a (i.e. make a the subject) from (i) or (ii); and the value of b from (ii) or (iii).

1. If $\frac{a}{b} = \frac{c}{d}$, what is the value of b? of c? of d?
2. If $\frac{p}{q} = \frac{r}{s}$, what is the value of p? of q? of r? of s?

The formula for finding the area of a triangle is $A = \frac{1}{2}bh$. This is the same as $\frac{A}{1} = \frac{bh}{2}$.

3. If $A = \frac{1}{2}bh$, what is the value of b? of h?
4. If in question 3 $b = 3$ and $h = 5$, what is A?
5. If in question 3 A is $17\frac{1}{2}$ and b is 5, what is h?
6. If in question 3 A is $31\frac{1}{2}$ and h is 7, what is b?
7. Make r^3 the subject of the formula $V = \frac{4}{3}\pi r^3$.
8. If $r^2 = 64$, what is r?
9. If $r^3 = 27$, what is r?
10. If $r^3 = ps$, what is r?
11. In question 7 make r the subject of the formula, and find its value when $V = 33{\cdot}5104$ ($\pi = 3{\cdot}1416$).

EXERCISE 52.—EQUATIONS WITH BRACKETS

First remove the brackets, then solve in the usual way. Check the result.

1. $8x - 2(3x + 2) = 2$.
2. $3(2x + 3) = 5(x + 2)$.
3. $5(x + 2) - 4(x + 2) = 0$.
4. $2(3x - 2) - 2(x - 1) + 3 = 3$.
5. $3(2x + 1) - 5(x - 1) = 2(x + 3) - 1$.
6. $4(3x - 2) - 2(5x - 4) = 7 - 3(x - 1)$.
7. $x - 5(2x - 10) + 4(x + 6) = 3(x + 8) + 2$.
8. $2x + 3(x - 3) = 2(x + 2) - (x - 5) - 6$.
9. $5(x - 2) - 3(2x - 1) = 5 - 3x - 2(x - 4)$.
10. $4x - 3(x - 5) + 5(x + 3) - 10 = 4(x + 4) + 12$.
11. $3x - 4(x + 1) + 3(2x - 3) = 5(2x - 1) - 3$.
12. $4(3x + 4) - 4(x + 2) = 3(3x - 5) - 3(2x - 1)$.

Consider this example :

$$(x + 1)(x + 4) = (x + 3)(x - 2)$$
$$\therefore\ x^2 + 5x + 4 = x^2 + x - 6.$$

Since x^2 appears on both sides, they cancel.

$$\therefore\ 5x + 4 = x - 6$$
$$5x - x = -6 - 4$$
$$4x = -10$$
$$x = -2\tfrac{1}{2}$$

Checking by substitution we find that each side of the original equation $= -\frac{9}{4}$.

Solve and check :

13. $(x + 2)(x - 2) = x(x - 1)$.
14. $(x - 1)(x + 3) = (x - 2)(x + 9)$.
15. $(x - 2)(x + 3) = (x - 1)(x + 1)$.
16. $(x - 3)(x - 4) = (x - 5)(x - 1)$.
17. $(x + 2)(x + 5) = (x + 4)^2$.
18. $(x - 4)(x + 6) = (x + 4)(x + 2)$.
19. $(2x - 1)(2x + 2) = (2x - 2)(2x + 5)$.
20. $(3x - 2)(x - 1) = (x - 2)(3x + 3)$.
21. $2(2x - 3)(3x - 7) = 12(x - 2)^2$.
22. $6(x - 4)^2 = (2x - 9)(3x - 10)$.
23. $(3x + 4)(2x + 5) = 6(x + 6)^2$.

EASY TEST 9

1. Solve the following equations: (i) $x + 7 = 15$, (ii) $x - 9 = 21$, (iii) $3x = 21$.
2. Solve the equations and check your answers:
   (i) $3x + 7 - 2 = 2x + 5 + 3$.
   (ii) $2x - 5 + 5x - 2 = 3x + 12 - 9 + 6$.
   (iii) $x - 4 + 6x - 2 = 5x - 7 + 3$.
3. Solve and check:
   (i) $14 + 8 - 1 = 4x - 2x + 5x$.
   (ii) $4x + 5 = 2x - 7$.
   (iii) $2x + 3 - 5x = x + 15$.
4. If $ab = cd$, what is the value of a?
5. If $ab = cd$, what is the value of d?
6. Change the formula $d = vt$ so that v becomes the subject.
7. Make t the subject of the formula $d = vt$.
8. If $\frac{w}{x} = \frac{y}{z}$, what is the value of wz?
9. If $\frac{w}{x} = \frac{y}{z}$, what is the value of w?
10. Solve and check:
    (i) $2(x + 3) = 8$.
    (ii) $5x - (2x + 4) = 2$.
    (iii) $4(x - 2) - 2(x - 3) = x + 3$.
11. Simplify: $\left(\frac{xy}{b^2}\right)\left(\frac{ab}{x^2y^2}\right)$.
12. Divide $9x^2 - 16y^2$ by $3x + 4y$.
13. Resolve $5x^2 - 7x - 6$ into factors.
14. If $a = 5$, $b = 4$, $c = 2$, $d = 0$, find the value of:
    $$\frac{bc^2}{4} + \frac{ab^2}{c} - \frac{cd^2}{a} - \frac{ac^2}{b}.$$
15. If a room is x ft. long and $(x - 3)$ ft. wide, what is the area of the floor?
16. Resolve into factors: (i) $2x^2 + x - 1$, (ii) $3x^2 + x - 2$.
17. Simplify $\frac{2x^2 + x - 1}{3x^2 + x - 2}$.

1. Solve : (i) $6x = 72$, (ii) $\frac{x}{3} = 5$, (iii) $\frac{x}{4} = \frac{3}{5}$.
2. Solve these equations and check your answers :
   (i) $7 + 6x - 5 - 3x = 5x - 4 + 3x + 1$.
   (ii) $4x - 5 + 2x - 1 = 3 + x + 6 + 2x$.
   (iii) $5x - 1\frac{1}{2} - 2x - 2\frac{1}{2} = 3x + 1\frac{1}{4} - x + \frac{3}{4}$.
3. Solve and check :
   (i) $7x + 9 = 3 + 4x$.
   (ii) $5x + 3 - 8x - 2 = 6 - 2x$.
   (iii) $2x - 2\frac{1}{2} - 6x = 4\frac{3}{4} - 3x + 1\frac{3}{4}$.
4. If $3ab = cd$, what is the value of a ?
5. If $xy^2 = 4mn$, what is the value of m ?
6. Change the formula $s = \frac{1}{2}ft^2$ so that f is the subject.
7. Make t the subject of the formula $s = \frac{1}{2}ft^2$.
8. If $\frac{m}{n} = \frac{p}{q}$, what is the value of n ?
9. If $\frac{3x}{4y} = \frac{5p}{2q}$, what is the value of q ?
10. Solve and check :
    (i) $4(x + 2) - 3(x + 3) = 4$.
    (ii) $2x - 6(x - 4) = 4(x - 2)$.
    (iii) $5x - 3(x - 7) = 6 - 4(x - 3)$.
11. Simplify : $\frac{3(a - b)}{4(a + b)^2} \times \frac{10(a + b)}{6(a - b)^2}$.
12. Divide $\frac{9}{16}x^2 - 1$ by $\frac{3}{4}x - 1$.
13. Resolve $20a^2 - a - 12$ into factors.
14. If $x = 6$, $y = 4$, $z = 2$, $w = 0$, find the value of :
    $$\frac{xy^2}{z} - \frac{xz^2}{y} - \frac{w^2x}{z} + \frac{xyz}{12}.$$
15. If the area of a square is $(4x^2 + 12x + 9)$ sq. ft., what is the length of its side ?
16. A tent pole 12 ft. high is held up by ropes tied to pegs in the ground 5 ft. away from the foot of the pole. How long is each of these ropes ?
17. If $p = q - 1$, find the value in terms of p of the expression $q^2 + 2q - 1$.

EXERCISE 53.—H.C.F. I

The factors of 12 are 1, 2, 3, 4, 6, 12.

The factors of 20 are 1, 2, 4, 5, 10, 20.

1, 2, and 4 are the common factors of 10 and 20.

4 is the highest common factor of 10 and 20.

Since 1 and the number itself are factors of every number, they are generally left out of account; but we cannot leave the number itself out of account in finding the H.C.F. For example, the H.C.F. of 6 and 18 is not 3, but 6.

The H.C.F. of $6a^2bc$ and $8ab^2cd$ is $2abc$.

The H.C.F. of $5x$ and $15x^2y$ is $5x$.

Find the H.C.F. of the following:

1. ab, abc.
2. a^2b, ab^2.
3. $4x^2y$, $6xy^2z$.
4. $5mnpr$, $6mn$.
5. $6abc^2$, $9b^2c^2$.
6. 14, $7ab$.
7. $3ab$, $6ac$, $12ad$.
8. $10xyz$, $6y^2z$, $14yz^2$.
9. $2mn$, $5m^2n^2$, $7mnp$.
10. $4r^2sp$, rs^2p, $5sp^2$.
11. $3ab$, $6a^2bc$, $9abcd$.
12. $6ab^2$, $21ab^2c^2$, $15ab^2x$.

Note that while the brackets are still there $(a + b)$ is a single term.

So is $(a + b)(x - y)$; so is $(a + b)^2(x + y)^3(m + n)$.

In finding the H.C.F. of the following do not untie the bundles.

13. $(a + b)^2$; $(a + b)(a - b)$; $(a + b)(a^2 + b^2)$.
14. $10(x + y)(x-y)$; $15(x + y)^2(x-y)$; $25(x + y)(x-y)^2$.
15. $(m + n)^4(m-n)^4$; $(m + n)^3(m-n)^2$; $(m + n)^2(m-n)^3$.
16. $6a^2b(a - b)^3$; $9ab^2(a - b)^2$; $12b^3(a - b)^4$.
17. $4(2a + b)(2a - b)$; $6(a + 3b)(2a - b)^2$; $8(2a - b)(3a + 4b)$.
18. $20(p + q)(p - q)$; $15(p - q)^2(p^2 + q^2)$; $25(p - q)(p^2 + 2pq + q^2)$.
19. $xy^2(x + y)(x - 2y)$; $x^2y(x + y)(x - y)(x - 2y)$; $x^2y^2(x + y)^2(x - 2y)^3$.
20. $4x^2y^2(x + 2)(x - 1)^2$; $8xy^3(x + 3)(x - 1)$; $6x^3y(x + 2)(x + 3)(x - 1)$.
21. $12a^2bc(a + b)(a - b)(a^2 + b^2)$; $6ab^2c(a - 2b)(a^2 + b^2)$; $9abc^2(a^2 + b^2)(a - b)$.

EXERCISE 54.—H.C.F. II

Find the H.C.F. of $2a + 4b$ and $3a + 6b$.

$$2a + 4b = 2(a + 2b)$$
$$3a + 6b = 3(a + 2b)$$

The H.C.F. is $a + 2b$.

Find the H.C.F. of the following :

1. $6a - 3b,\ 2a - b$.
2. $2a + 2b,\ 5a + 5b$.
3. $2x + 2y - 2z,\ 4x + 4y - 4z$.
4. $a + b,\ a^2 + 2ab + b^2$.
5. $3a + 3b,\ a^2 - b^2$.
6. $2x - 2y,\ x^2 - y^2$.
7. $ax - ay,\ x^2 - 2xy + y^2$.
8. $x^2 + 5x + 6,\ x^2 + 6x + 8$.
9. $6m^2 - 5mn + n^2,\ 2m^2 - 7mn + 3n^2$.
10. $a^2 - 2ab + b^2,\ a^2 - b^2,\ 5a - 5b$.
11. $2x^2 + 9x + 10,\ 2x^2 + 11x + 15,\ 2x^2 + 13x + 20$.
12. $3x^2 - 7x + 2,\ 6x^2 - 5x + 1,\ 3x^2 - 10x + 3$.
13. $3x^2 - x - 2,\ 6x^2 + x - 2,\ 3ax + 2a$.
14. $4a^2 - 9b^2,\ 2a^2 - ab - 3b^2,\ 6a^2 - 5ab - 6b^2$.
15. $9m^2 + 6mn + n^2,\ 18m^2 - 2n^2,\ 3m^2 - 8mn - 3n^2$.
16. $4x^2 - 1$; $4x^2 + 4x + 1$; $2x^2 - 3x - 2$.
17. $p^2 - q^2$; $p^2 - 2pq + q^2$; $p^2 - 3pq + 2q^2$.
18. $x^2 - 2x + 1$; $6x - 6$; $x^2 - 1$.
19. $3a^2b + 3ab^2$; $6a^3b + 6a^2b^2$; $a^2 + 2ab + b^2$; $ac + ad + bc + bd$.
20. $3a^2b + 3ab^2$; $6a^3b + 6a^2b^2$; $9a^2 + 18ab + 9b^2$; $3ac + 3ad + 3bc + 3bd$.
21. $x^2 - 2xy + y^2$; $x^2 - y^2$; $4x^2 - 4xy$; $ax - ay + bx - by$.
22. $6x^2 - 12xy + 6y^2$; $8x^2 - 8y^2$; $4x^2 - 4xy$; $10ax - 10ay + 10bx - 10by$.
23. $ac + ad + bc + bd$; $ad + ae + bd + be$; $af + ag + bf + bg$; $ah + ak + bh + bk$.
24. $ac + ad - bc - bd$; $ad + ae - bd - be$; $af + ag - bf - bg$; $ah + ak - bh - bk$.
25. $6abx + 6aby$; $9ax^2 - 9ay^2$; $12abx^2 + 24abxy + 12aby^2$; $9amx + 9amy + 9anx + 9any$.

The multiples of 6 are: 6, 12, 18, 24, 30, 36, 42, 48, 54, etc. without end.

The multiples of 9 are: 9, 18, 27, 36, 45, 54, etc. without end.

The common multiples of 6 and 9 are: 18, 36, 54, etc. without end.

The least common multiple (or L.C.M.) of 6 and 9 is 18.

We have to regard a number as a multiple of itself, just as we have to regard it as a factor of itself. It is itself multiplied by 1.

Hence the L.C.M. of 6 and 18 is 18.

The L.C.M. of $4a^2b$, $6ab^2$, and $8abc$ is $24a^2b^2c$. It is the least number that is exactly divisible by all of them.

Find the L.C.M. of the following:

1. ab, abc.
2. $a, ab, abc, abcd$.
3. a^2b, ab^2, a^2b^2.
4. $xy, 5xy, x^2y, xyz$.
5. $3mn, 4mnp, 3\ mnp^2$.
6. $12pr, 20p^2r^2, 15p^3r^3$.
7. $3ab, 6ac, 12ad$.
8. $4x^2yz, 12xy^2z, 18xyz^2$.
9. $6, p^2qr, pqr, 7qr^2$.
10. $10abc, 5bcd, 15cde$.
11. $4x, 6xy^2, 4z^2$.
12. $12abx, 9bcy, 15cdz$.

Sometimes it is necessary to factorize first. Keep the factors in your answers.

13. $a^2 + ab, a^2 + 2ab + b^2$.
14. $a + b, c + d$.
15. $x^2 - y^2, x^2 - 2xy + y^2$.
16. $6m - 3n, 4m^2 - n^2, 2m^2 + mn - n^2$.
17. $a + b, a^2 - 2ab + b^2, a^2 - b^2$.
18. $p, r, 3p - 3r, p^2 + 2pr + r^2, p^2 - r^2$.
19. $a, ab, abc, abc + b^2c, ac + bc, ab + b^2$.
20. $x^2, xy, y^2, x^2y^2, x^2y - xy^2, x^3 - 2x^2y + xy^2,$ $x^2y + 2xy^2 + y^3$.
21. $2x, y, 2z^2, x^2y^2, 2x^2 - 2y^2, x + y, x^2 - y^2, 2x + 2y,$ $x^2 + 2xy + y^2$.
22. $3ab, 4ac, 6bc, 2a^2b + 2ab^2, abc + abd, 2ac^2 + 2acd,$ $4a^2c + 4abc$.
23. $8, pq, pr, qr, pqr, 3p + 3q, 6p - 6q, 12p^2 - 12q^2,$ $2p^2 + 2pq, 3pq - 3q^2$.

EXERCISE 56.—FRACTIONS

$$\frac{1}{2} + \frac{1}{3} = \frac{3+2}{2 \times 3} = \frac{5}{6}.$$

$$\frac{1}{a} + \frac{1}{b} = \frac{b+a}{ab} \text{ or } \frac{a+b}{ab}.$$

This cannot be further simplified, as we do not know the exact arithmetical values of a and b.

Deal with fractions in algebra as you would in arithmetic.

$$\frac{a}{b} - \frac{c}{d} = \frac{ad-bc}{bd}.$$

A more difficult example :

$$\frac{a}{a+b} + \frac{b}{a-b} = \frac{a(a-b)+b(a+b)}{(a+b)(a-b)}$$

$$= \frac{a^2-ab+ab+b^2}{a^2-b^2} = \frac{a^2+b^2}{a^2-b^2}.$$

Simplify the following :

1. $\frac{h}{3} + \frac{h}{4}.$
2. $\frac{h}{4} - \frac{h}{6}.$
3. $\frac{r}{2} + \frac{r}{5} - \frac{3r}{10}.$
4. $\frac{2t}{3} + \frac{3t}{2} - \frac{t}{6}.$
5. $\frac{a+1}{2} + \frac{a+2}{4}.$
6. $\frac{2b+1}{2} - \frac{b-1}{3} + 4.$
7. $\frac{1}{x} - \frac{1}{y}.$
8. $\frac{5}{x} - \frac{5}{y}.$
9. $\frac{a}{x} - \frac{a}{y}.$
10. $\frac{5}{a-b} - \frac{4}{a+b}.$
11. $\frac{m-n}{m+n} + \frac{m+n}{m-n}.$
12. $\frac{m+n}{m-n} - \frac{m-n}{m+n}.$
13. $\frac{1}{a} + \frac{1}{b} + \frac{1}{c}.$
14. $\frac{a}{a-b} - \frac{a^2}{a^2-b^2}.$
15. $\frac{p}{p+q} - \frac{p^2}{p^2+2pq+q^2}.$
16. $\frac{x}{y} - \frac{y}{x} + \frac{xy}{x+y}.$

EXERCISE 57.—EQUATIONS WITH FRACTIONS

When fractions occur in an equation the first step towards solving it is to get rid of the fractions by multiplying both sides by the L.C.M. of the denominators, e.g.:

$$\frac{x+1}{2} + \frac{x-1}{4} - \frac{x+2}{6} = 4.$$

Multiply by 12:

$$\begin{aligned} 6(x+1) + 3(x-1) - 2(x+2) &= 48 \\ 6x + 6 + 3x - 3 - 2x - 4 &= 48 \\ 7x - 1 &= 48 \\ 7x &= 48 + 1 \\ 7x &= 49 \\ x &= 7 \end{aligned}$$

Check. Left side $= \frac{8}{2} + \frac{6}{4} - \frac{9}{6} = 4 + 1\frac{1}{2} - 1\frac{1}{2} = 4.$

Right side $= 4.$

Solve the following equations and check each answer:

1. $x + \frac{x}{5} = 12.$
2. $\frac{3x}{4} = \frac{5}{6}.$
3. $\frac{2x}{3} = 1.$
4. $\frac{4y}{7} = \frac{2}{3}.$
5. $n + \frac{n}{3} = 4.$
6. $\frac{t}{2} - \frac{t}{3} = 2.$
7. $\frac{2n-5}{3} = 5.$
8. $\frac{t-2}{3} = \frac{t-3}{4}.$
9. $\frac{3x}{2} - 4 = \frac{2x}{3} - \frac{2}{3}.$
10. $\frac{n+1}{2} - \frac{n+2}{3} = 1.$
11. $\frac{2y-1}{3} - \frac{y-1}{4} = y - 3.$
12. $\frac{x+4}{5} - 2x = \frac{x-1}{10} - 20.$
13. $\frac{t}{2} - \frac{t}{4} + \frac{t-6}{3} = 1 + \frac{t}{3}.$
14. $\frac{x-2}{2} - \frac{x-1}{4} = 3 - x.$
15. $\frac{x+2}{3} - \frac{x+1}{4} = \frac{11}{24}.$
16. $\frac{x+3}{2} - \frac{x-1}{3} - 2\frac{1}{12} = 0.$

EXERCISE 58.—DECIMAL COEFFICIENTS

Example (i): $0{\cdot}5x = 0{\cdot}035$

$$\therefore \quad x = \frac{\cdot 035}{\cdot 5} = \frac{\cdot 35}{5} = \cdot 07, \text{ or } 0{\cdot}07.$$

Check: $\quad 0{\cdot}5 \times 0{\cdot}07 = 0{\cdot}035.$

Note: The 0 in the units place may be dropped in working.

Example (ii): $1{\cdot}5(n + 2) - 0{\cdot}8(n - 2) = 6{\cdot}7$

$$\begin{aligned} 1{\cdot}5n + 3 - \cdot 8n + 1{\cdot}6 &= 6{\cdot}7 \\ \cdot 7n + 4{\cdot}6 &= 6{\cdot}7 \\ \cdot 7n &= 6{\cdot}7 - 4{\cdot}6 \\ \cdot 7n &= 2{\cdot}1 \\ n &= 3 \end{aligned}$$

Check: $1{\cdot}5 \times 5 - \cdot 8 \times 1 = 7{\cdot}5 - \cdot 8 = 6{\cdot}7.$

A still simpler way is to multiply both sides by some power of ten (such as 10; or 100; or 1,000), which would rid the equation of decimal fractions. The above example could be worked thus:

$$1{\cdot}5(n + 2) - 0{\cdot}8(n - 2) = 6{\cdot}7$$

Multiply by 10: $\quad 15(n + 2) - 8(n - 2) = 67$

By simplifying this: $\quad n = 3$

Solve the following equations and check your answers:

1. $0{\cdot}06x = 1{\cdot}2.$
2. $0{\cdot}01 + 3x = 4{\cdot}51.$
3. $\dfrac{0{\cdot}4}{n} = 0{\cdot}08.$
4. $0{\cdot}3(n - 1) + 0{\cdot}2 = 0{\cdot}8.$
5. $0{\cdot}5(n - 1) = 0{\cdot}4(n + 2) - 0{\cdot}9.$
6. $\dfrac{4x - 5}{2x + 7} = 0{\cdot}1.$
7. $\dfrac{3x - 2}{4x} = 0{\cdot}55.$
8. $\dfrac{x + 0{\cdot}8}{3} + \dfrac{x - 0{\cdot}2}{2} - 2 = 0.$
9. $\dfrac{3x - 0{\cdot}1}{4} = \dfrac{2x - 0{\cdot}4}{2}.$
10. $0{\cdot}01(x + 1) + 0{\cdot}1(x - 1) = 1{\cdot}01.$
11. $\dfrac{4x - 1}{6x - 0{\cdot}5} = \dfrac{1}{4}.$
12. $3(r + 1) - 2(5r - 1) = 2{\cdot}2.$

EXERCISE 59.—PROBLEMS INVOLVING ONE UNKNOWN

(i) I think of a number, multiply it by 3, and subtract 6. The remainder is 27. What was the original number ?

Let x = the original number.

$3x - 6 = 27.$

By solving this easy equation we find that $x = 11$. Check by seeing if it satisfies the conditions of the problem.

(ii) The formula for the perimeter of a rectangle is $p = 2(l + b)$. If the perimeter of a rectangular flower-bed is 42 ft. and its length is 3 ft. more than twice its breadth, find its length and its breadth.

$$l = 2b + 3 \quad . \quad . \quad . \quad . \quad \text{(i)}$$

By substituting for p and l in the formula, only one unknown is left.

Thus :
$$42 = 2(2b + 3 + b)$$
$$42 = 2(3b + 3)$$

By solving this equation we find that $b = 6$, and by substituting for b in (i) we find that $l = 15$.

(iii) Find a pair of numbers whose sum is 37 and whose difference is 15.

Let n = one of the numbers.

$37 - n$ = the other number.

$$n - (37 - n) = 15$$

By simplification: $n = 26$

The pair of numbers is 26 and 11.

Make equations for the following problems, then solve them and check your answers :

1. I am thinking of a number. When it is doubled and 18 is added the total is 50. What is the number ?
2. A man is 6 years older than his wife. Forty years ago he was twice as old as she then was. How old is he now ?
3. The area of a triangle is given by the formula $A = \frac{1}{2}bh$, where b means the number of units of length in the base and h the number in the height. What is the height of a triangle the area of which is $7\frac{1}{2}$ sq. in. and the base 3 in. ?

4. What is the length of the base of a triangle whose area is $246\frac{1}{2}$ sq. ft. and whose height is 17 ft.?
5. Find a number such that if you add 20 to a third of it you get the same result as if you added 15 to half of it.
6. The circumference of a circle can be found from the formula $c = \pi d$, where d means the length of the diameter. Write the formula for finding the perimeter of a semicircular area of radius r. Find the perimeter of a semicircular area whose radius is 3 in. (Make a diagram. Take $\pi = \frac{22}{7}$.)
7. There are 30 coins in a bag, consisting of half-crowns and florins. Together they amount to £3 6*s.* How many coins were there of each kind?
8. In the formula $RM = EL$, find the value of M in terms of R, E, and L. What is the value of E when $R = 8$, $M = 15$, and $L = 20$?
9. Divide a line 32 in. long into two parts, such that one may be three-fifths of the other.
10. A bankrupt owes B twice as much as he owes A, and C as much as he owes A and B together. Out of £240, which is to be divided among them, what should each receive?
11. Divide £24 among two persons so that for every shilling one receives the other may receive five.
12. How much money is there in a purse when a third part and a fourth part together amount to £7?
13. A company of 304 persons consists of men, women, and children. There are four times as many men as children, and three times as many women as children. How many of each are there?
14. A person spends one-third of his income on board and lodging and one-tenth in clothing, and has £340 left. What is his income?
15. Find that number, the third part of which added to its fifth part makes 24.

1. What is the H.C.F. of $6x^2$ and $9xy$?
2. What is the H.C.F. of $4(a + b)$, $6(a + b)^2$, $8(a + b)(a - b)$?
3. Find the H.C.F. of $3p - 3q$, $4p^2 - 4q^2$.
4. Find the L.C.M. of $3a^2b$, $4ab^2$, $6abc$.
5. What is the L.C.M. of $p^2 + pq$, $p^2 + 2pq + q^2$?
6. Simplify $\frac{2}{p} + \frac{3}{q}$.
7. Simplify $\frac{n}{2} + \frac{n}{3} - \frac{n}{6}$.
8. Simplify $\frac{x + y}{x - y} - \frac{x - y}{x + y}$.
9. Solve $\frac{3x}{4} = \frac{5}{6}$.
10. Solve $\frac{x + 2}{4} - 2 = \frac{x + 3}{3} - 3$.
11. Solve $0{\cdot}7x - 0{\cdot}8 = 0{\cdot}2x + 0{\cdot}7$.
12. Find a number the half of which exceeds the third by 7.
13. Simplify by removing brackets—
$$5x^2 - (4x + 2) - (3x^2 - x) + 3.$$
14. Factorize $5a^2 + 10ab + 5b^2$.
15. Factorize $4a^3 - 8a^2b + 4ab^2$.
16. Factorize $2x^2 - 2y^2$.
17. Factorize $2x^2 + 10x + 12$.
18. Simplify $\frac{(a + b)^2 - (a - b)^2}{a^2 + b^2}$.
19. If $\frac{p}{q^2} = \frac{x^2}{y}$, what is the value of y ?
20. If $x^2 = 81$, what is the value of x ?
21. A packet of paper x in. high contains y sheets. How thick is each sheet ?
22. A rug is l ft. long and w ft. wide. What is its area (i) in square feet, (ii) in square yards ?
23. A man of 36 has a son of 10. In how many years' time will the father be twice as old as his son ?

1. What is the H.C.F. of $10a^3b^3$, $15a^2b^4$, and $25a^4b^2$?
2. What is the H.C.F. of $4(x - y)$ and $6(x^2 - 2xy + y^2)$?
3. What is the H.C.F. of $6m + 6n$, $12m^2 - 12n^2$, $9m^2 + 18mn + 9n^2$?
4. Find the L.C.M. of $0{\cdot}2x^3$, $0{\cdot}8x^5$, $0{\cdot}4x^2$.
5. Find the L.C.M. of—
$3a - 3b$, $6a^2 - 6b^2$, $4a^2 - 8ab + 4b^2$.
6. Simplify $\frac{2a}{3} + \frac{3a}{2}$.
7. Simplify $\frac{x + y}{3} - \frac{x - y}{4} + \frac{x + y}{6}$.
8. Simplify $\frac{4}{a + b} - \frac{2}{a - b} + \frac{3}{a^2 - b^2}$.
9. Solve $\frac{5}{6x} = \frac{3}{20}$.
10. Solve $\frac{x + 3}{2} - \frac{x - 4}{3} = \frac{x - 2}{5} + 3$.
11. Solve $0{\cdot}4x - 1{\cdot}2 = 0{\cdot}1x + 0{\cdot}3$.
12. Find a number the third of which exceeds the fourth by 8.
13. Simplify by removing brackets—
$2x^2 - [3x - (x^2 - 2)] - (2x + 6)$.
14. Factorize $3x^2 - 3y^2$.
15. Factorize $4a + 4b + ab + b^2$.
16. Factorize $ax - ay + bx - by$.
17. Factorize $12x^2 - 2x - 24$.
18. Simplify $\frac{(a + b)^2 - 2ab}{a^2 + b^2}$.
19. If $x^2 = 144$, what is the value of x ?
20. If $\frac{x^2}{y^2} = \frac{a}{b}$, what is the value of x ?
21. The difference of the squares of two consecutive numbers is 13. Find the numbers.
22. The sum of two numbers is 140, and their difference is equal to one-third of the greater. Find the numbers.

EXERCISE 60.—PERFECT SQUARES

Look again at Exercises 26 and 32 and bear in mind that

(i) $a^2 + 2ab + b^2$ and

(ii) $a^2 - 2ab + b^2$

are perfect squares, (i) being $(a + b)^2$ and (ii) $(a - b)^2$. The only difference between (i) and (ii) is in the middle term, and you can remember what that term is from the phrase " twice their product."

What must be added to (ii) to make (i) ? Obviously $4ab$.

What must be subtracted from (i) to make (ii) ? $4ab$.

What must be added to the following to make them perfect squares ?

1. $x^2 + 2xy$.
2. $4x^2 + 12xy$.
3. $a^2 + 10ab$.
4. $24mn + 16n^2$.
5. $9p^2 + 16s^2$.
6. $a^2x^2 + b^2y^2$.
7. $x^2 + xy + y^2$.
8. $4a^2 + 10ab + 9b^2$.
9. $24x^2 + 20xy + 4y^2$.
10. $9m^2 + 12mn + 3n^2$.
11. $16a^2 + 21ab + 8b^2$.
12. $2r^2 + 10rs + 7s^2$.
13. $x^2 - 2xy$.
14. $25x^2 - 30xy$.
15. $- 10mn + 25n^2$.
16. $9m^2 - 13mn + 4n^2$.
17. $4p^2 - 24pq + 25q^2$.
18. $8x^2 - 25xy + 14y^2$.

What must be subtracted from the following to make them perfect squares ?

19. $a^2 - 5ab + 9b^2$.
20. $25m^2 + 44mn + 16n^2$.
21. $10m^2 - 12mn + 4n^2$.
22. $19x^2 - 23xy + 9y^2$.
23. $25p^2 - 18pq + 5q^2$.
24. $11x^2 - 9xy + 6y^2$.

Write down the squares of the following :

25. $2a + \frac{1}{2}$.
26. $z - \frac{1}{4}$.
27. $mn + \frac{1}{3}$.
28. $x + \dfrac{1}{y}$.
29. $y - \dfrac{1}{z}$.
30. $c - \dfrac{1}{c}$.

Find the square root of the following :

31. $m^2 + \dfrac{2m}{n} + \dfrac{1}{n^2}$.
32. $\dfrac{1}{m^2} - \dfrac{2n}{m} + n^2$.
33. $\dfrac{1}{x^2} - \dfrac{2}{xy} + \dfrac{1}{y^2}$.
34. $x^2 + 2 + \dfrac{1}{x^2}$.

EXERCISE 61.—REGULAR EXPRESSIONS

Four thousand and seven = 4,007.

Algebraically it may be expressed as $4x^3 + 7$. Here there is no need to put noughts to indicate missing terms, otherwise the expression would be—

$$4x^3 + 0x^2 + 0x + 7.$$

Note that the powers of x gradually diminish, and in the last term disappear (or $= x^0$, which is always 1).

Now look at this expression :

$$a^3 + a^2b + ab^2 + b^3.$$

Observe (i) that the powers of a gradually diminish, (ii) that the powers of b gradually increase, and (iii) that every term has three dimensions, for the expression may be written—

$$aaa + aab + abb + bbb.$$

There is no place in this *regular* expression for a term of two dimensions, such as a^2, or ab, or b^2.

$$a^3 + b^3 = a^3 + 0a^2b + 0ab^2 + b^3.$$

It is an advantage to put it like this when doing long multiplication or long division.

To simplify $\dfrac{a^3 + b^3}{a + b}$ divide $a^3 + 0a^2b + 0ab^2 + b^3$ by $a + b$, as in arithmetical long division. The quotient is $a^2 - ab + b^2$.

$$\frac{a^3 + b^3}{a + b} = a^2 - ab + b^2, \text{ and}$$

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$

Express the following in full, using 0 as the coefficient of the missing terms :

1. $a^2 + b^2$. 2. $a^2 - b^2$. 3. $a^3 - b^3$.
4. $a^4 + b^4$. 5. $a^5 + b^5$. 6. $a^6 - b^6$.

Answer the following :

7. $a + b$ is a factor of $a^5 + b^5$; what is the other factor ?
8. Divide (i) $a^3 - b^3$ by $a - b$, (ii) $a^5 - b^5$ by $a - b$, (iii) $a^4 - b^4$ by $a + b$, and (iv) $a^4 - b^4$ by $a - b$.
9. Write out the quotient of $a^7 - b^7$ divided by $a - b$.

Examine these series of binomials, or expressions of two terms :

(i) $a + b, a^3 + b^3, a^5 + b^5, a^7 + b^7$, etc. with odd indices.

(ii) $a - b$, $a^3 - b^3$, $a^5 - b^5$, $a^7 - b^7$, etc. with odd indices.

(iii) $a^2 + b^2$, $a^4 + b^4$, $a^6 + b^6$, $a^8 + b^8$, etc. with even indices.

(iv) $a^2 - b^2$, $a^4 - b^4$, $a^6 - b^6$, $a^8 - b^8$, etc. with even indices.

The first binomial in series (i) is divisible by $a + b$, and so is every other one in the series.

The first binomial in series (ii) is divisible by $a - b$, and so is every other one in the series.

The first binomial in series (iii) is not divisible by either $a + b$ or $a - b$; neither is any other one in the series.

The first binomial in series (iv) is divisible by both $a + b$ and $a - b$; so is every other one in the series.

Look at these two quotients :

$$\frac{a^5 + b^5}{a + b} = a^4 - a^3b + a^2b^2 - ab^3 + b^4.$$

$$\frac{a^5 - b^5}{a - b} = a^4 + a^3b + a^2b^2 + ab^3 + b^4.$$

Note that when the divisor is $a + b$ the signs in the quotient are alternately $+$ and $-$; and when the divisor is $a - b$ the signs in the quotient are all $+$.

Note that $n^2 - 1$ is of the form $a^2 - b^2$, for it may be written $n^2 - 1^2$. Similarly $8x^3 + 27y^3$ is of the form $a^3 + b^3$, for it may be written $(2x)^3 + (3y)^3$.

1. Without actually dividing, write out the quotient of $x^3 + 1$ divided by $x + 1$.
2. $x - 1$ is a factor of $x^3 - 1$; what is the other factor ?
3. $x + 1$ is a factor of $x^5 + 1$; what is the other factor ?
4. When $x^4 - 1$ is divided by $x + 1$ what is the quotient ?
5. Multiply x^2 by x^3.
6. Simplify x^2x^2, i.e. $(x^2)^2$.

7. Simplify (i) $x^2x^2x^2$, i.e. $(x^2)^3$, (ii) x^3x^3, i.e. $(x^3)^2$.
8. Remembering that $a^2 - b^2 = (a + b)(a - b)$, factorize (i) $4m^2 - 9n^2$, (ii) $16x^2 - 25y^2$.

Commit to memory :

$$\mathbf{a^3 + b^3 = (a + b)(a^2 - ab + b^2)}$$
$$\mathbf{a^3 - b^3 = (a - b)(a^2 + ab + b^2)}$$

Factorize the following :

9. $n^3 + 1$.
10. $n^3 - 1$.
11. $8x^3 + 27y^3$.
12. $8x^3 - 27y^3$.
13. $27p^3 + 1$.
14. $27p^3 - 1$.
15. One factor of $8x^3 - 27$ is $2x - 3$; what is the other ?
16. One factor of $a^3 + 64$ is $a + 4$; what is the other ?

Find the products of these factors :

17. $(p + q)(p^2 - pq + q^2)$.
18. $(p + 2q)(p^2 - 2pq + 4q^2)$.
19. $(2p - q)(4p^2 + 2pq + q^2)$.
20. $(2b + 3c)(4b^2 - 6bc + 9c^2)$.
21. $(2b - 3c)(4b^2 + 6bc + 9c^2)$.
22. $(3x + 1)(9x^2 - 3x + 1)$.
23. $(x - 3)(x^2 + 3x + 9)$.
24. $(x + 5)(x^2 - 5x + 25)$.
25. $(5x - 1)(25x^2 + 5x + 1)$.
26. $(2a - 5b)(4a^2 + 10ab + 25b^2)$.
27. $(5a + b)(25a^2 - 5ab + b^2)$.
28. $(3p + 5q)(9p^2 - 15pq + 25q^2)$.
29. $(5p - 3q)(25p^2 + 15pq + 9q^2)$.
30. $(x + 6)(x^2 - 6x + 36)$.
31. $(6x - 1)(36x^2 + 6x + 1)$.
32. $(2a + 7b)(4a^2 - 14ab + 49b^2)$.
33. $(7a - 2b)(49a^2 + 14ab + 4b^2)$.
34. $(\frac{1}{2}a + b)(\frac{1}{4}a^2 - \frac{1}{2}ab + b^2)$.
35. $(\frac{1}{2}a - b)(\frac{1}{4}a^2 + \frac{1}{2}ab + b^2)$.
36. $(a + \frac{1}{2}b)(a^2 - \frac{1}{2}ab + \frac{1}{4}b^2)$.
37. $(a - \frac{1}{2}b)(a^2 + \frac{1}{2}ab + \frac{1}{4}b^2)$.

Factorize the following :

38. $r^2 - s^2$.
39. $r^3 - s^3$.
40. $4x^2 - 1$.
41. $8x^3 - 1$.
42. $9m^2 - 4n^2$.
43. $27m^3 - 8n^3$.
44. $\frac{1}{4}b^2 - c^2$.
45. $\frac{1}{8}b^3 - c^3$.
46. $\frac{1}{8}b^3 + c^3$.

EXERCISE 63.—FACTORS V

Verify these results by multiplying out as rapidly as you can:

$$(a + b)^2 = a^2 + 2ab + b^2$$
$$(a - b)^2 = a^2 - 2ab + b^2$$
$$(a + b)(a - b) = a^2 - b^2$$
$$(a + b)(a^2 - ab + b^2) = a^3 + b^3$$
$$(a - b)(a^2 + ab + b^2) = a^3 - b^3$$

You should practise working these until you can do the whole in three minutes.

Simplify the following:

1. $(a + b)(a^2 - ab + b^2) + (a - b)(a^2 + ab + b^2)$.
2. $(x + y)(x^2 - xy + y^2) - (x - y)(x^2 + xy + y^2)$.
3. $(a + b)^2 - \dfrac{a^3 + b^3}{a + b}$.
4. $\dfrac{m^3 + n^3}{m + n} + (m + n)^2$.
5. $\dfrac{p^3 + q^3}{p + q} + (p - q)^2$.
6. $(p - q)^2 - \dfrac{p^3 + q^3}{p + q}$.
7. $(p - q)^2 - \dfrac{p^3 - q^3}{p - q}$.
8. $(2a + b)^2 - \dfrac{8a^3 + b^3}{2a + b}$.
9. $\dfrac{27x^3 - 1}{3x - 1} - (3x - 1)^2$.
10. $\dfrac{x^3 + 8}{x + 2} - (x - 2)^2$.
11. $\dfrac{a^2 + 2ab + b^2}{a + b} + \dfrac{a^2 - 2ab + b^2}{a - b} - 2(a - b)$.
12. $\dfrac{4(x^2 + 2x + 1)}{x + 1} - \dfrac{3(x^2 - 2x + 1)}{x - 1} + 7(x - 1)$.
13. $\dfrac{x^6 - y^6}{(x + y)(x^2 - xy + y^2)} - \dfrac{x^6 - y^6}{(x - y)(x^2 + xy + y^2)}$.
14. $\dfrac{m^4 - n^4}{m^2 + n^2} - \dfrac{m^4 - n^4}{m^2 - n^2} + 2(m + n)(m - n)$.
15. $(p + q)(p^2 - pq + q^2) + (p + q)(p - q) - p^3 - q^3$.

Factorize the following:

16. $8a^3 + 27b^3$.
17. $8a^3 - 27b^3$.
18. $\frac{1}{8}a^3 + \frac{1}{27}b^3$.
19. $16x^2 - 49y^2$.
20. $\frac{1}{9}p^2 + \frac{1}{6}pq + \frac{1}{16}q^2$.
21. $\frac{9}{16}p^2 - p + \frac{4}{9}$.

EASY TEST 11

1. What must be added to $x^2 + y^2$ to make it a perfect square ?
2. What must be subtracted from $x^2 + y^2$ to make it a perfect square ?
3. What must be added to $16m^2 - 8m$ to make it a perfect square ?
4. Fill in the missing terms of $x^3 + y^3$.
5. Factorize $x^3 - y^3$.
6. One factor of $a^3 + 27$ is $a + 3$; what is the other factor ?
7. Factorize $8p^3 - 64q^3$.
8. Simplify $\dfrac{y^2 - 1}{y + 1} - (y - 1)$.
9. Simplify $\dfrac{(x + y)^2}{x^3 + y^3}$.
10. Simplify $\dfrac{5 + a}{2} - \dfrac{8 - a}{4}$.
11. What value of x will satisfy the equation—
$$3x - \frac{x - 4}{2} = 5(x + 2)\ ?$$
12. Solve $0{\cdot}5x + 0{\cdot}6 = 1{\cdot}1$.
13. If $0{\cdot}037x = 4$, find the value of x correct to 1 decimal place.
14. Factorize $6x^2 + 12xy + 6y^2$.
15. Factorize $mx - my + nx - ny$.
16. Factorize $3m - 3n - pm + pn$.
17. Simplify $\dfrac{x^2 + 4x + 3}{x^2 + 5x + 6}$.
18. Reduce $\dfrac{a^3 + x^3}{a^2 - x^2}$ to its lowest terms.
19. Divide $\dfrac{2a + 3b}{c + d}$ by $\dfrac{c - d}{2a - 3b}$.
20. Find the value of $\left(a - \dfrac{x^2}{a}\right)\left(\dfrac{a}{x} + \dfrac{x}{a}\right)$.

1. What must be added to $9x^2 + 20xy + 14y^2$ to make it a perfect square ?
2. What must be taken from $25p^2 - 8p + 2$ to make it a perfect square ?
3. What must be added to $\frac{1}{4}a^2 + \frac{1}{4}b^2$ to make it a perfect square ?
4. Fill in the missing terms of $x^4 + y^4$.
5. Factorize $x^3 + y^3$.
6. One factor of $\frac{1}{8}x^3 - 1$ is $\frac{1}{2}x - 1$; what is the other factor?
7. Factorize $27m^3 + 64n^3$.
8. Simplify $\dfrac{a^2 - b^2}{a + b} + \dfrac{a^3 - b^3}{a - b}$.
9. Simplify $\dfrac{p^3 - q^3}{p - q} - \dfrac{p^3 + q^3}{p + q}$.
10. Simplify $\dfrac{3x - y}{2a} - \dfrac{x - 3y}{a}$.
11. What value of x will satisfy the equation—
$$4x - \frac{2x - 5}{4} - \frac{x + 2}{5} = 2\tfrac{1}{2}\ ?$$
12. Solve $0{\cdot}4x - 0{\cdot}35 = 0{\cdot}2x + 0{\cdot}5$.
13. If $3{\cdot}1416x = 1$, find the value of x correct to 3 places of decimals.
14. Factorize $0{\cdot}5x^2 + xy + 0{\cdot}5y^2$.
15. Factorize $12a^2b + 11ab^2 - 15b^3$.
16. Factorize $a^2bx + a^2by - ab^2x - ab^2y$.
17. Divide $x - \dfrac{1}{x}$ by $1 + \dfrac{1}{x}$.
18. Simplify $\dfrac{4}{n^2 - 1} - \dfrac{3}{n + 1} + \dfrac{2}{n - 1}$.
19. Divide $1 - \dfrac{b^4}{a^4}$ by $\dfrac{a}{b} + \dfrac{b}{a}$.
20. Reduce $\dfrac{6x^2 + 7x - 3}{9x^2 + 3x - 2}$ to its lowest terms.
21. The perimeter of an equilateral triangle is x in. On the same base a square is drawn. What is : (i) its area ; (ii) its perimeter ?

EXERCISE 64.—SIMULTANEOUS EQUATIONS

Look at problem (iii) of Exercise 59.

Find a pair of numbers whose sum is 37. There are many such pairs.

Find a pair of numbers whose difference is 15. There are many such pairs.

But there is only one pair which have 37 as their sum and *at the same time* have 15 as their difference. The word *simultaneous* means *at the same time*. Simultaneous equations have at least two unknowns.

Let us call one of the pair of numbers m, and the other n; and let us bear in mind that if equals are added to equals the sums are equal, and if equals are subtracted from equals the remainders are equal.

$$
\begin{array}{lrl}
 & \text{(i)}\ m + n &= 37 \\
 & \text{(ii)}\ m - n &= 15 \\
\text{Add:} & 2m &= 52 \\
 & m &= 26 \\
\text{Subtract:} & 2n &= 22 \\
 & n &= 11
\end{array}
$$

The two numbers are 26 and 11, and no other two numbers will satisfy both equation (i) and equation (ii).

Having found by addition that $m = 26$, I could find n by substituting 26 for m in either equation (i) or equation (ii), e.g.:

$$
\begin{aligned}
\text{(i)}\ 26 + n &= 37 \\
n &= 37 - 26 \\
n &= 11
\end{aligned}
$$

In the following examples find one of the unknown numbers by addition or subtraction and then find the other by substitution. Check your answer in the equation you did *not* use for substituting.

1. $3x + y = 9.$
   $2x - y = 1.$
2. $2x - 3y = 1.$
   $2x + 5y = 25.$
3. $5x + 2y = 9.$
   $5x - y = 3.$
4. $x + 3y = 13.$
   $5x - 3y = 11.$

5. $5m + n = 11.$
   $7m + n = 13$

6. $3m - 2n = 5.$
   $6m - 2n = 14.$

7. $4x - 3y = 0.$
   $7x - 3y = 9.$

8. $4x - 6y = 6.$
   $x + 6y = 9.$

9. $r = 2 + s.$
   $r = 8 - s.$

10. $n = 6 - p.$
    $n = p + 2.$

11. $2(t - v) = 6.$
    $3t - 2v = 13.$

12. $4(a - b) = 12.$
    $4a = 5b + 10.$

If the equations contain fractions, the first step should be to get rid of the fractions by multiplying *both sides* by the L.C.M. of the denominators.

Thus :

$$\frac{2x}{3} - \frac{3y}{2} = 2\tfrac{1}{2} \quad . \quad . \quad \text{(i)}$$

$$\frac{x}{3} + \frac{5y}{4} = 3\tfrac{1}{4} \quad . \quad . \quad \text{(ii)}$$

Multiply (i) by 6 : $4x - 9y = 15 \quad . \quad . \quad \text{(iii)}$

Multiply (ii) by 12 : $4x + 15y = 39 \quad . \quad . \quad \text{(iv)}$

Subtract (iii) from (iv) : $24y = 24$

$y = 1$

Substitute 1 for y in (iii) : $4x - 9 = 15$

$4x = 15 + 9 = 24$

$x = 6$

Caution : You cannot get rid of fractions by multiplying one side only of an equation, you must multiply both sides completely by the same number.

13. $\frac{x}{3} + \frac{5y}{2} = 12.$
    $\frac{2x}{7} - y = 4.$

14. $\frac{5x}{3} - \frac{y}{2} = 3.$
    $2x + \frac{3y}{4} = 9.$

15. $\frac{2x}{5} + \frac{7y}{3} = 18.$
    $\frac{3x}{5} + \frac{3y}{2} = 15.$

16. $\frac{2x}{3} + \frac{5y}{7} = 30.$
    $\frac{4x}{5} + \frac{3y}{4} = 33.$

EXERCISE 65.—SIMULTANEOUS EQUATIONS: ADDITION AND SUBTRACTION

When it is not possible to eliminate (i.e. get rid of) one of the unknowns by the addition or subtraction of the equations as they stand, it can be done by adjusting the equations thus:

$$\begin{array}{lrcll}
 & 2x - 3y & = & 2 & \quad \ldots \text{(i)} \\
 & 3x + 2y & = & 16 & \quad \ldots \text{(ii)} \\
\text{Multiply (i) by 3:} & 6x - 9y & = & 6 & \quad \ldots \text{(iii)} \\
\text{Multiply (ii) by 2:} & 6x + 4y & = & 32 & \quad \ldots \text{(iv)} \\
\text{Subtract (iii) from (iv):} & 13y & = & 26 & \\
 & y & = & 2 & \\
\text{Substitute 2 for } y \text{ in (i):} & 2x - 6 & = & 2 & \\
 & 2x & = & 2 + 6 & \\
 & 2x & = & 8 & \\
 & x & = & 4 &
\end{array}$$

Check: Substitute 4 for x and 2 for y in (ii).

$$12 + 4 = 16$$

If the equations are not in the form (i) and (ii) above they should be reduced to that form before further steps are taken, e.g.:

$$\begin{array}{lrcll}
 & 3(x + 2) & = & 4(y - 2) & \quad \ldots \text{(i)} \\
 & y & = & 5(x - 1) & \quad \ldots \text{(ii)} \\
\text{Simplify (i):} & 3x + 6 & = & 4y - 8 & \\
 & 3x - 4y & = & -14 & \quad \ldots \text{(iii)} \\
\text{Simplify (ii):} & y & = & 5x - 5 & \\
 & -5x + y & = & -5 & \quad \ldots \text{(iv)}
\end{array}$$

Now deal with (iii) and (iv) and see if you get $x = 2$ and $y = 5$.

Begin by multiplying (iv) by 4 and eliminating y.

Solve the following and check your answers. Alter the original equations as little as possible. It is often sufficient to alter only one of them.

1. $4x - 3y = 9.$
   $2x + 5y = 11.$
2. $3x + y = 15.$
   $4x - 3y = 7.$
3. $3x + 2y = 8.$
   $5x - 6y = 4.$
4. $x + 2y = 11.$
   $4x - 5y = 5.$

Exer. 65. SIMULTANEOUS EQUATIONS: ADDITION AND SUBTRACTION

5. $3x - 2y = 2.$
   $2x + 3y = 23.$

6. $5x - y = 1.$
   $3x + 2y = 11.$

7. $5x - 3y = 21.$
   $2x + 6y = -6.$

8. $2x + 3y = 1.$
   $5x - 4y = 37.$

9. $3x + 4y = 10.$
   $5x + 3y = 2.$

10. $7x - 3y = 13.$
    $3x - 2y = 7.$

11. $3(x - y) = 12.$
    $4(x + y) = 32.$

12. $4x - 30 = 7 - 5y.$
    $9x - 4 = 2x + 3y + 2.$

13. $\frac{x}{4} = \frac{y}{5}.$
    $3(x - 6) = 2(y - 6).$

14. $\frac{x+2}{3} = \frac{y+4}{4}.$
    $5x - 2(y - 2) = 30.$

15. $\frac{x}{2} = \frac{y-2}{5}.$
    $3(x - 2) = 2(y - 9).$

16. $\frac{x+4}{3} = y - 3.$
    $5(x - 5) = 3(y - 2).$

17. $2x + 3y = 43.$
    $10x - y = 7.$

18. $5x + 7y = 43.$
    $11x + 9y = 69.$

19. $\frac{x}{5} + \frac{y}{6} = 18.$
    $\frac{x}{2} - \frac{y}{4} = 21.$

20. $\frac{x}{3} + \frac{y}{4} = 9.$
    $\frac{x}{4} + \frac{y}{5} = 7.$

21. $\frac{x}{2} + \frac{y}{3} = 1.$
    $\frac{x}{3} + \frac{y}{4} = 1.$

22. $\frac{x}{2} + \frac{y}{3} = 18.$
    $\frac{x}{5} + \frac{y}{4} = 10.$

23. $x + \frac{y}{3} = 15.$
    $\frac{x}{4} + y = 12.$

24. $\frac{x}{2} - \frac{y}{5} = 4.$
    $\frac{x}{7} + \frac{y}{3} = 7.$

25. $x - \frac{y}{5} = 1.$
    $\frac{x}{3} + \frac{y}{2} = 6.$

26. $\frac{x}{6} + \frac{y}{4} = 0.$
    $\frac{x}{4} + y = 5.$

27. $x - \frac{y}{9} = 3.$
    $\frac{x}{5} + y = 19.$

28. $\frac{x}{5} - \frac{y}{4} = 5.$
    $\frac{x}{7} - \frac{y}{2} = 1.$

29. $\frac{x}{5} + \frac{y}{3} = 7.$
    $\frac{x}{3} - \frac{y}{4} = 2.$

30. $\frac{x}{2} - \frac{y}{12} = 5.$
    $\frac{x}{7} + \frac{y}{8} = 5.$

EXERCISE 66.—SIMULTANEOUS EQUATIONS: SUBSTITUTION

There is another way to solve simultaneous equations; it begins by taking the simpler of the two equations and finding the value of one unknown in terms of the other, thus:

$$2x + 3y = 6 \quad . \; . \; . \; \text{(i)}$$
$$5x + 7y = 16 \quad . \; . \; . \; \text{(ii)}$$

From (i):
$$2x = 6 - 3y$$
$$x = \frac{6 - 3y}{2} \quad \text{(iii)}$$

Substitute for x in (ii): $\dfrac{5(6 - 3y)}{2} + 7y = 16$

Multiply by 2:
$$5(6 - 3y) + 14y = 32$$
$$30 - 15y + 14y = 32$$
$$-15y + 14y = 32 - 30$$
$$-y = 2$$
$$y = -2$$

Substitute for y in (iii) (which = (i)): $x = \dfrac{6 + 6}{2} = 6$

$$\therefore \; x = 6, \; y = -2$$

Check by substituting these numbers for x and y in (ii).

Express x in terms of y in the following:

1. $2x = 3y$.
2. $5x = 2y - 1$.
3. $3x - 2 = y$.
4. $3(x - 4) = 5y$.
5. $\dfrac{x}{2} = \dfrac{y}{3}$.
6. $\dfrac{x}{4} = 3(5y - 1)$.

Express y in terms of x in the following:

7. $5x - 3y = 0$.
8. $3(x - 4) = 2y$.
9. $\dfrac{x - 1}{2} = \dfrac{y - 3}{4}$.
10. $\dfrac{x - 5}{3} = 2(y + 4)$.
11. $\dfrac{3x + 1}{2} = 7y$.
12. $\dfrac{x - 5}{3} = \dfrac{2(y - 2)}{5}$.

Solve the following by substitution. Select the easier of the two equations and from it choose the unknown which is easier to express in terms of the other unknown. For instance, if (3) above were one of the equations, it

would be easier to express y in terms of x than to express x in terms of y.

13. $3x + 2y = 21.$
    $2x - 3y = 1.$

14. $5x = 2(y - 1).$
    $3x + 4 = 7y - 32.$

15. $\frac{m}{3} - 2 = n - 6.$
    $2(m - 7) = \frac{n}{4} + 8.$

16. $\frac{m}{5} + \frac{n}{3} = 6.$
    $2m - 3(m - n) = 12.$

17. $\frac{1}{2}p - \frac{1}{2}q = 1.$
    $3p = 4q + 3.$

18. $\frac{2x + 3}{3} = y + 1.$
    $5(x + 6) = 4(y + 6) - 1.$

19. $0{\cdot}5r - 0{\cdot}4s = 0.$
    $0{\cdot}75r - 0{\cdot}3s = 1{\cdot}5.$

20. $\frac{1}{3}(r + s) = \frac{1}{2}.$
    $\frac{1}{4}(r - s) = \frac{1}{8}.$

21. $\frac{t + 1}{5} + \frac{v - 1}{2} = 4.$
    $\frac{t - v}{3} = 1\frac{1}{3}.$

22. $\frac{2t - v}{5} = t - v.$
    $\frac{t}{2} + \frac{v}{3} = 6.$

23. $2a - 5 = 3(b + 8).$
    $\frac{1}{5}(3a - 1) = b + 9.$

24. $\frac{y + 7}{4} + \frac{1}{2} = 15 + 2z.$
    $y - z = 2.$

Sometimes simultaneous equations appear in this form:

$$3x - 2y = 2x - y - 1 = 3.$$

Arrange thus:

$$\begin{aligned} 3x - 2y &= 3 \quad . \ . \ \text{(i)} \\ 2x - y - 1 &= 3 \\ 2x - y &= 4 \quad . \ . \ \text{(ii)} \end{aligned}$$

Solve the following either by the method of addition and subtraction or by the method of substitution, whichever you find easier.

25. $5x - 3y = 4x + y - 8 = 11.$
26. $3x - 8 = 2x - 3(y - 1) = 7.$
27. $3x + 2y - 1 = 5x - 3y = 4x - y + 1.$
28. $\frac{x + 5}{5} = \frac{x - y}{3} = 4.$
29. $4(m + n) - 7 = 3(m - n) + 5 = 17.$
30. $\frac{1}{2}(p - q) + 3 = \frac{1}{3}(p + 2q) + 1 = 6.$

EXERCISE 67.—PROBLEMS INVOLVING TWO UNKNOWNS

Example I. A number consists of two digits the sum of which is 9. If the digits are reversed, the number will be larger by 45. What is the number ?

Let x = the first digit
and y = the second digit.

Then the value of the number is $10x + y$, and its value when the digits are reversed is $10y + x$.

We now have two equations showing different relations between x and y :

$$\text{(i)} \quad x + y = 9$$
$$\text{(ii)} \quad 10y + x = 10x + y + 45.$$

Solving these simultaneous equations we find that $x = 2$ and $y = 7$. We check the result by finding that it satisfies the conditions of the problem.

Example II. A greengrocer serves a customer with 3 lb. of apples and 4 lb. of pears and charges 3*s*. He serves another customer with 4 lb. of apples and 3 lb. of pears and charges 2*s*. 10*d*. What is the price per pound of the apples and the pears respectively ?

Let x = the number of pence paid for a pound of apples and y = the number of pence paid for a pound of pears.

$$\text{(i)} \quad 3x + 4y = 36.$$
$$\text{(ii)} \quad 4x + 3y = 34.$$

Solving these simultaneous equations we find that the apples cost 4*d*. a pound and the pears 6*d*. a pound.

Solve the following :

1. Find two numbers whose sum is 40 and whose difference is 6.
2. A number of two digits is 9 times as great as the sum of its digits, and exceeds by 63 the number formed by making the digits change places. What is the original number ?
3. I am thinking of a number of two digits. If 36 is added to it, the digits will be reversed. If 9 is subtracted from it, the remainder will equal the sum of the two digits. What is the number ?

4. If 5 lb. of butter and 3 lb. of cheese cost 11*s*. 2*d*., and 2 lb. of butter and 4 lb. of cheese cost 7*s*. 6*d*., what is the price per pound of each commodity ?
5. Find the price of petrol per gallon and of oil per quart if 12 gallons of petrol and a quart of oil cost £1, and 5 gallons of petrol and a pint of oil cost 8*s*. 6*d*.
6. In 3 years' time a father will be $3\frac{1}{2}$ times as old as his son. Two years ago he was 6 times as old as his son. What are their present ages ?
7. Two kinds of tickets were issued for a concert, one kind for the front seats and the other for the back seats. When 160 front-seat tickets and 400 back-seat tickets were sold the total takings amounted to £60, but when the concert was repeated and 120 front-seat tickets were sold and 500 back-seat tickets, the takings were £65. What was the price of a front-seat ticket, and what the price of a back-seat ticket ?
8. The length of a rectangular flower-bed is 3 ft. longer than its breadth. Its perimeter is 54 ft. Find its length and its breadth.
9. What fraction is that which becomes equal to $\frac{3}{4}$ when its numerator is increased by 9, and equal to $\frac{1}{2}$ when its denominator is diminished by 2 ? (Let x = the numerator and y = the denominator.)
10. A and B together possess £305. If A's money were three times what it really is, and B's five times what it really is, the sum would be £1,165. What money does each possess ?
11. If Tom's money were increased by 17*s*. he would have three times as much as Dick ; but if Dick's money were diminished by 5*s*. he would have half as much as Tom. Find the sum possessed by each.
12. For 30*s*. I can buy either 15 lb. of butter and 5 lb. of cheese, or 9 lb. of butter and 15 lb. of cheese. Find the price of a pound of each.

EASY TEST 12

1. Solve $x + 3y = 5.$
$x - y = 1.$

2. Solve $3x + y = 5.$
$x + 3y = 7.$

3. Solve $4x - 3y = 6.$
$2x + 5y = 16.$

4. Solve $2x + 3y = 9.$
$3x - 2y = 7.$

5. Solve $4x - 3y = 14.$
$3x - 5y = 5.$

6. Solve $2x + 3y - 3 = 10.$
$5x - 1 = 2y + 3.$

7. Solve $\frac{x}{2} + \frac{y}{3} = 8.$
$\frac{x}{3} + \frac{y}{2} = 7.$

8. Solve $x + \frac{y}{3} = 6.$
$y + \frac{x}{4} = 7.$

9. Find a pair of numbers whose sum is 43 and whose difference is 13.
10. Solve $5x - 4y = 3x + 2y - 10 = 8.$
11. Simplify $\frac{x + 4}{2x-3} \div \frac{x^2 + 8x + 16}{4x^2 - 9}.$
12. Simplify $6p + 4q - \{3p - 2(q + 1)\}.$
13. If $a + b = 7$, what is the value of—
$$a^2 + 2ab + b^2 + 2a + 2b\ ?$$
14. Simplify $(2p - q)(4p^2 + 2pq + q^2).$
15. If $a = 0{\cdot}8$ and $b = 1{\cdot}2$, what does $5a^2 - 3ab$ equal ?
16. If $x = y - 2$, find in terms of y the value of—
$$x^2 - 2x - 6.$$
17. Write the quotient of $a^6 - b^6$ divided by $a - b$.
18. A man buys x eggs for $5s.$ and sells them at a profit at $2d.$ each. What is his profit in pence ?
19. The angles of a triangle measured in degrees are $x + 3$, $x + 5$, and $2x$. What are their numerical values ?
20. What value of n will make $2n + 6$ and $3n - 1$ equal ?
21. To a number I add twice the next larger number. The result is 38. What is the original number ?
22. If $5s = 4 + 3s$, and $3t = 15 - 2t$, find the value of :
(i) st ; (ii) $s + t$.
23. Find two numbers such that twice the first plus half the second is equal to 21, and half the first plus three times the second is equal to 34.

1. Solve $5x - 3y = 13.$
$2x + 3y = 1.$

2. Solve $x + 5y = 3.$
$3y - 2x = 7.$

3. Solve $4x - 3y = 7.$
$5x - 4y = 8.$

4. Solve $2(x - 1) + 3y = 9.$
$3x - 2(y + 1) = 8.$

5. Solve $4x - (y + 3) = 8.$
$5x - 2(y + 6) = 1.$

6. Solve $2x - \frac{1}{3}y = 7.$
$\frac{1}{2}x + 3y = 11.$

7. Solve $\dfrac{p}{3} + \dfrac{q}{2} = 6.$
$\dfrac{p}{2} - \dfrac{q}{4} = 1.$

8. Solve $\dfrac{2}{x} + \dfrac{5}{y} = \dfrac{19}{xy}.$
$\dfrac{3}{x} - \dfrac{1}{y} = \dfrac{3}{xy}.$

9. The angles of a triangle are x, y, and 60 degrees respectively. $x = \frac{1}{2}y$. Find the angles x and y.

10. Solve $5x + 2(y + 3) + y = 3x - (x - 5) + 2y + 10 = 17.$

11. Simplify $\dfrac{(2a + b)(a^2 - 9b^2)}{(2a^2 + 7ab + 3b^2)}.$

12. Simplify $\dfrac{pq(p^2 - q^2)(p^2 + pq + q^2)(p - q)}{q(p^3 - q^3)(p + q)}.$

13. If $a - b = 6$, what is the value of—

$$4a - 4b + \frac{a^3 - b^3}{a^2 + ab + b^2}\ ?$$

14. Simplify $(2m + n)(2m - n)(4m^2 + n^2)$.

15. John and Henry play at marbles. After John has won a third of Henry's marbles, each has 12. How many had each to begin with ?

16. If $a = 2b = 3c = 4d = 12$, what is the value of :

$$3a - 2(b - c) - 2d\ ?$$

17. The sum of the ages of n boys is p years. What will be the sum of their ages in 3 years' time ?

18. How must a piece of string 6 in. long be divided so that one part may be 0·6 of the other part ?

19. Simplify $\dfrac{t}{0{\cdot}1} + \dfrac{t}{0{\cdot}2} - \dfrac{t}{0{\cdot}3}.$

20. If the perimeter of a rectangle is p inches, and its length is l inches, what is its breadth ?

EXERCISE 68.—FACTORS VI

Test the truth of the following by multiplying out, and then memorize the results.

$$(\mathbf{a} + \mathbf{b})^3 = \mathbf{a}^3 + 3\mathbf{a}^2\mathbf{b} + 3\mathbf{a}\mathbf{b}^2 + \mathbf{b}^3$$
$$(\mathbf{a} - \mathbf{b})^3 = \mathbf{a}^3 - 3\mathbf{a}^2\mathbf{b} + 3\mathbf{a}\mathbf{b}^2 - \mathbf{b}^3$$

Factorize :

1. $3a^2 + 6ab + 12ac$.
2. $8x^2y - 12xy^2 + 20xyz$.
3. $4(p + q) - 3(p + q) + 8(p + q)$.
4. $3m^2 + 6mn + 3n^2$.
5. $ax - 5a - x^2 + 5x$.
6. $4p^2 - 8pq + 4q^2$.

Find the following products :

7. $(a + b)(a + c)$.
8. $(x + z)(m + n)$.
9. $(a + 2)(b + c)$.
10. $(x + 2)(x - 5)$.
11. $(a - b)(d - f)$.
12. $(ax + b)(cx - d)$.
13. $(x + 3)(x + 7)$.
14. $(x - 3)(x - 8)$.
15. $(b - 4)(b + 3)$.
16. $(2x + 1)(3x + 1)$.
17. $(5x + 3)(x - 4)$.
18. $(3x - 5)(2x + 4)$.
19. $(2a + 3b)(5a - 2b)$.
20. $(\frac{1}{2}x + y)(\frac{1}{2}x + z)$.
21. $(mn - 2)(mn + 6)$.
22. $\left(\frac{1}{x} + \frac{2}{y}\right)\left(\frac{2}{x} - \frac{3}{y}\right)$.
23. $(a + m)(a - m)$.
24. $(\frac{1}{2}p + \frac{1}{4}q)(\frac{1}{2}p - \frac{1}{4}q)$.
25. $\{(a+b)+c\}\{(a+b)-c\}$.
26. $(3a + 2b)^2$.
27. $(3a + 2b)^3$.

Factorize :

28. $x^2 - p^2$.
29. $16y^2 - 9b^2$.
30. $9x^2 - 12xy + 4y^2$.
31. $(a - b)^2 - c^2$.
32. $9x^2 - (y + z)^2$.
33. $(a + b)^2 - (x + y)^2$.
34. $m^3 + 3m^2n + 3mn^2 + n^3$.
35. $m^3 + n^3$.
36. $p^3 - 3p^2q + 3pq^2 - q^3$.
37. $8x^3 - 1$.
38. $8a^3 + 27b^3$.
39. $15x^2 - x - 6$.

Simplify :

40. $\dfrac{a^2 + 2ab + b^2}{a + b} - \dfrac{a^3 - b^3}{a - b}$.
41. $(x + y)^3 - (x - y)(x^2 + xy + y^2)$.
42. $(x + y)(x^2 - xy + y^2) - (x - y)^3$.
43. $\dfrac{x^2 - 7x + 12}{x^2 - 9}$.
44. $\dfrac{x + 4}{x + 3} - \dfrac{x + 3}{x + 4}$.

EXERCISE 69.—EQUATIONS DIFFICULT TO CHECK

Try to solve this equation :

$$\frac{x}{4} - \frac{5(x-3)}{7} = \frac{1}{3}(5x-1)$$

The usual way to begin is by multiplying both sides by the L.C.M. of the denominators, that is by 84. If you work correctly you will find $x = 1\frac{29}{179}$. If you try to check this result by substitution you will find it much more troublesome than the original working, and much more likely to contain errors. It is therefore better to look over your working again carefully, or to solve the equation by different steps. For example, the equations could be arranged thus :

$$\frac{x}{4} - \frac{5x-1}{3} = \frac{5(x-3)}{7}$$

This simplifies to : $\dfrac{-17x+4}{12} = \dfrac{5x-15}{7}$

By simplifying this it is again found that $x = 1\frac{29}{179}$, which confirms the first result.

Solve the following equations, and check your answers as recommended above :

1. $\dfrac{3x+4}{5} = \dfrac{4x-7}{3}$.
2. $\dfrac{5x+7}{6} = 8x - 3$.
3. $5(3x-1) = \dfrac{4x+3}{6}$.
4. $\dfrac{x-7}{5} = \dfrac{5x-4}{7}$.
5. $\dfrac{3x+5}{12} = 4(7x+8)$.
6. $\frac{5}{6}x - 4(x+3) = \frac{3}{5}x - 3(x-5)$.

Find the value of the following unknown numbers to two places of decimal :

7. $3x = 9x - 4$.
8. $4(3x+5) = 5(x+7)$.
9. $5(4n-3) = 6(2n-3)$.
10. $\dfrac{5t-8}{3} = \dfrac{4t+5}{5}$.
11. $\dfrac{3v+4}{5} = 2v + 9$.
12. $\frac{2}{3}(4r+9) = \frac{1}{5}(7r-3)$.

EXERCISE 70.—SQUARE ROOT I

Our knowledge of algebra helps us to find square root in arithmetic. We know that the square root of $a^2 + 2ab + b^2$ is $a + b$. We could set this out somewhat like a division sum.

$$\begin{array}{r|l}
 & a^2 + 2ab + b^2(a + b \\
 & a^2 \\
\hline
2a + b & \quad 2ab + b^2 \\
 & \quad \underline{2ab + b^2}
\end{array}$$

$(a+b)^2 = (a+b)(a+b)$

$a^2 + 2ab + b^2$

We know that $56^2 = 3{,}136$, and that therefore

$$\sqrt{3{,}136} = 56$$

But $56^2 = (50 + 6)^2 = 50^2 + 2 \times 50 \times 6 + 6^2$, and this could be arranged as in the algebra example above.

It could also be set out like this :

$$\begin{array}{r|l}
 & 3{,}136\ (50 + 6 \\
 & \underline{2{,}500} \\
100 + 6 & \ 636 \\
 & \ \underline{636}
\end{array}
\qquad \text{or,} \qquad
\begin{array}{r|l}
 & 3{,}136\ (56 \\
 & 25 \\
106 & 636 \\
 & \underline{636}
\end{array}$$

Remembering that $10^2 = 100$, $100^2 = 10{,}000$, $1{,}000^2 = 1{,}000{,}000$, we can see the truth of this table :

Root		*Number*
1 digit	. . .	1 or 2 digits
2 digits	. . .	3 or 4 digits
3 digits	. . .	5 or 6 digits
4 digits	. . .	7 or 8 digits

In other words, there are twice as many (or twice as many — 1) digits in a number as in its square root. We can therefore find the number of digits in the square root by pairing the digits in the original number, working from the right. Thus :

The square root of $\overline{49}$ has 1 digit.
The square root of $7\overline{84}$ has 2 digits.
The square root of $\overline{70}\,\overline{56}$ has 2 digits.
The square root of $1\overline{56}\,\overline{25}$ has 3 digits.
The square root of $\overline{19}\,\overline{44}\,\overline{81}$ has 3 digits.

How many digits are there in the square roots of the following numbers ?

1. 81. 2. 169. 3. 2,304. 4. 20,736.
5. 893,025. 6. 46,294,416.

Now suppose we have to find the square root of 784.

1*st Step*. Pair the digits, thus $7\overline{84}$.

2*nd Step*. Find the square root of the first part, 7 (which is really 700). Since 2 squared is 4, and 3 squared is 9, we know that the square root lies between 20 and 30, and that the first of its two digits is 2.

3*rd Step*. Subtract 2^2 (which by its place value is 20^2) to find how much of the number remains to be dealt with. We see that 384 remains.

$$\begin{array}{r|l} & 7\overline{84}(28 \\ & 4 \\ \hline 48 & 384 \\ & 384 \\ \hline \end{array}$$

4*th Step*. Use as a divisor double the part of the root already found, plus the new part, which is found to be 8.

5*th Step*. Complete as in long division.

Find by these steps the square root of the following :

7. 676. 8. 169. 9. 225. 10. 2,304. 11. 1,089.
12. 6,241. 13. 9,801. 14. 3,364. 15. 1,156.

$\left(\frac{3}{4}\right)^2 = \frac{3}{4} \times \frac{3}{4} = \frac{3^2}{4^2} = \frac{9}{16}$. Therefore $\sqrt{\frac{9}{16}} = \frac{\sqrt{9}}{\sqrt{16}} = \frac{3}{4}$.

As this is true of all fractions, $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$.

Find the square root of each of the following. First turn the mixed numbers into improper fractions.

16. $\frac{49}{81}$. 17. $\frac{225}{289}$. 18. $\frac{3,136}{9,801}$. 19. $\frac{1,521}{2,209}$.
20. $1\frac{15}{49}$. 21. $2\frac{82}{121}$. 22. $2\frac{743}{1,369}$. 23. $11\frac{353}{841}$.

EXERCISE 71.—SQUARE ROOT II

Following the same steps as in the last exercise try to work out for yourself the square root of 53,824, and then compare your working with the following :

$$\begin{array}{r|l}
 & \overline{53},\overline{824}(232 \\
 & 4 \\
43 & 138 \\
 & 129 \\ \hline
462 & 924 \\
 & 924 \\ \hline
\end{array}$$

In long division you bring down one digit at a time ; in square root you bring down two.

Find the square root of each of the following numbers :
1. 45,369. 2. 19,881. 3. 50,625. 4. 488,601.

We can often guess the square root of a number of three or four digits, if we know for certain that the number is a perfect square. If, for example, 1,521 is a perfect square, we know that its square root has two digits and that the first of them is 3, for 1,521 is between 30^2 (which is 900) and 40^2 (which is 1,600). We see, too, from the table on p. 63 that, the units figure being 1, the second figure in the square root must be either 1 or 9, and the square root must be either 39 or 31. Since 1,521 is much nearer 40^2 than 30^2, the correct root must be 39.

Find by inspection the square root of the following perfect squares :
5. 2,916. 6. 5,625. 7. 6,561. 8. 3,364.

The numbers we have so far considered are perfect squares ; but most numbers we have to deal with are not perfect squares, and we can only approximately find their square root ; but we can get nearer and nearer the true root by using decimals. What, for instance, is the value of $\sqrt{3}$? It is evidently greater than 1, for $1^2 = 1$; and not so great as 2, for $2^2 = 4$. Since, however, 3 is nearer to 4 than it is to 1, we can assume that its square root is larger than 1·5. Let us assume that it is 1·6.

Squaring 1·6 we get 2·56, which is short of 3. So 1·6 is not large enough. Let us try 1·7. Squaring 1·7 we get 2·89, which again is too small. Let us try 1·8. The square of this is 3·24, and this is too large. We infer, therefore, that $\sqrt{3}$ lies somewhere between 1·7 and 1·8.

We can more easily discover this by following the usual steps and bringing down two decimal places at a time, thus:

$$
\begin{array}{r|l}
 & 3\cdot\overline{00}\,\overline{00}\,\overline{00}(1\cdot732 \\
 & 1 \\
27 & 200 \\
 & 189 \\
343 & 1100 \\
 & 1029 \\
3462 & 7100 \\
 & 6924 \\
 & 176
\end{array}
$$

$$\sqrt{3} = 1\cdot732\ \ldots$$

We can go on like this as long as we like, but as the next digit in the root will be less than 5, we say that 1·732 is correct to three places. If the next figure had been 5 or more we should have regarded 1·733 as the correct answer.

The square root of a number which is not a perfect square is called a *surd.*

Find in the same way the value of the following surds to three decimal places:

9. $\sqrt{2}$. 10. $\sqrt{5}$. 11. $\sqrt{6}$. 12. $\sqrt{7}$.
13. $\sqrt{8}$. 14. $\sqrt{10}$. 15. $\sqrt{11}$. 16. $\sqrt{12}$.

Try to remember these results:

$\sqrt{2} = 1\cdot414\ \ldots$ or, approximately, $\frac{7}{5}$.

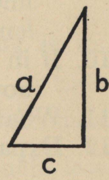
$\sqrt{3} = 1\cdot732\ \ldots$ or, approximately, $\frac{7}{4}$.

Find the square root of each of the following to one decimal place:

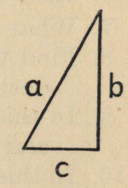
17. 20. 18. 68. 19. 153. 20. 200.
21. 300. 22. 18. 23. 32. 24. 48.

EASY TEST 13

1. Simplify $(m + n)^3 - (m - n)^3$.
2. Factorize $5x^2 - 5y^2$.
3. Factorize $x^2 - 12x + 35$.
4. Solve $\dfrac{3x + 5}{4} + 5 = 6$.
5. How many digits has the square root of 90,601 ?
6. 1,369 is a perfect square. What is its square root ?
7. What is the value of $\sqrt{7} \times \sqrt{7}$?
8. Find the square root of 30 correct to one place of decimal.
9. In this right-angled triangle express a in terms of b and c.
10. In this triangle express b in terms of a and c.
11. If $a = 13$ ft. and $c = 5$ ft., what is b ?
12. What is the square of $5\frac{3}{4}$?
13. Simplify $7x - (-5 + 3x) - 2(x + 1)$.
14. Simplify $\dfrac{1}{a - y} - \dfrac{2y}{a^2 - y^2}$.
15. Simplify $\dfrac{p + q}{x} \times \dfrac{y}{p^2 - q^2}$.
16. Solve $\dfrac{3}{4 - x} = \dfrac{2}{3 + x}$.
17. Area of circle $= \pi r^2$. Find the area of a circle whose radius is 7 ft. Take $\pi = 3\frac{1}{7}$.
18. Solve $3x + 2y = 9$.
    $5x + 3y = 14$.
19. Make R the subject of the formula $\dfrac{e}{W} = \dfrac{r}{R}$.
20. Solve $\dfrac{x + 1}{x - 2} = \dfrac{x + 3}{x - 5}$.
21. By what must $\dfrac{a}{b}$ be multiplied to give $\dfrac{b}{a}$?
22. The area of a rectangle is $5a^2$ sq. in., and its length is $10ab$ in. What is its breadth ?



1. Simplify $(2m + 1)^3 - (2m - 1)^3$.
2. Factorize $3x^4 - 3y^4$.
3. Factorize $(x + y)^2 - z^2$.
4. Solve $\dfrac{2x + 3}{2} - 4(x + 2) = 5$.
5. Find the square root of 200 to two places of decimal.
6. What is the value of $\sqrt{5} \times \sqrt{5} \times \sqrt{5}$?
7. What is the value of $(-7)^4$?
8. If a in this right-angled triangle $= 4$ in. and $b = 3$ in., find to two decimal places the length of c.

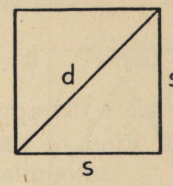


9. If in this triangle c is 2 in. and a 6 in., find to two places of decimal the length of b.
10. If in this triangle a is $(x + y)$ in. and b is $(x - y)$ in., how long is c?
11. What is the square of $8{\cdot}25$?
12. Expand $(0{\cdot}5x + 0{\cdot}4y)^2$.
13. Factorize $\dfrac{x^2}{16} - \dfrac{y^2}{25}$.
14. Factorize $x^3 - x^2 - 4x + 4$.
15. An expression can be resolved into two factors, each the sum of two numbers. One of the factors is $x + 4$. The whole expression has a value of 56 when $x = 4$. Find the second factor. Write down the complete expression before it was factorized.
16. Solve $\dfrac{4x - 0{\cdot}8}{2} = \dfrac{x}{0{\cdot}7} + 2x - 1{\cdot}4$.
17. The formula for the volume of a cone is $V = \frac{1}{3}\pi r^2 h$. Change it (i) so that h is the subject, (ii) so that r is the subject.
18. Solve $\dfrac{5x}{8} - \dfrac{2y}{3} = \dfrac{1}{4}$.

    $3x - 2y = 1$.
19. If $\dfrac{r}{s} = \dfrac{t}{v}$, what is the value of $\dfrac{s}{r} - \dfrac{v}{t}$?

EXERCISE 72.—ALGEBRA AND GEOMETRY

In this square s means the number of units in the side and d the number in the diagonal. The diagonal is the hypotenuse of the two right-angled triangles into which it divides the square.



$d^2 = s^2 + s^2 = 2s^2.$

$\therefore d = \sqrt{2s^2} = \sqrt{2}\sqrt{s^2} = \sqrt{2}s.$

Learn: $\mathbf{d = \sqrt{2}s}$; and since—

$\sqrt{2} = 1{\cdot}414$ approximately, $\mathbf{d = 1{\cdot}414s.}$

1. Find to the nearest tenth of an inch the diagonal of a square whose side is 3 in.
2. Find to the nearest tenth of a foot the diagonal of a square whose side is 120 ft.
3. If the side of a square is $5x$ ft., how long is its diagonal?

Since $d = \sqrt{2}s$, it follows that $s = \frac{d}{\sqrt{2}}$. As it is awkward to divide by a surd, and much less awkward to multiply by one, we transfer the surd from the denominator to the numerator by multiplying both numerator and denominator by the surd itself; or, which is the same thing, multiply the fraction by $\frac{\sqrt{2}}{\sqrt{2}}$:

$$\frac{d}{\sqrt{2}} = \frac{\sqrt{2}d}{\sqrt{2}\sqrt{2}} = \frac{\sqrt{2}d}{2}.$$

So instead of the formula $s = \frac{d}{\sqrt{2}}$ use $s = \frac{\sqrt{2}d}{2} = 0{\cdot}707d.$

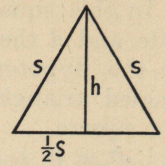
Find correct to two places of decimal:

4. The side of a square with a diagonal of 12 in.
5. The side of a square with a diagonal of 15 ft.
6. The side of a square with a diagonal of $10x$ ft.

Let us now find the relation between the height and the side of an equilateral triangle.

$$h^2 = s^2 - (\tfrac{1}{2}s)^2 = s^2 - \tfrac{1}{4}s^2 = \tfrac{3}{4}s^2$$

$$\therefore h = \sqrt{\frac{3}{4}s^2} = \frac{\sqrt{3}s}{2}.$$



Learn : $\mathbf{h} = \dfrac{\sqrt{3}\mathbf{s}}{2}$; and since—

$\sqrt{3} = 1{\cdot}732$ approximately, $\mathbf{h} = {\cdot}\mathbf{866s}$.

7. Find to the nearest tenth of an inch the height of an equilateral triangle of 4-in. side.
8. If the base of an equilateral triangle is 7 in., what is its height ?
9. Find to two places of decimal the height of an equilateral triangle if its side is 5 ft.

If we make s the subject of the formula $h = \dfrac{\sqrt{3}s}{2}$ we get $s = \dfrac{2h}{\sqrt{3}}$, and in order to get rid of the surd in the denominator we multiply the fraction by $\dfrac{\sqrt{3}}{\sqrt{3}}$ and find that $\mathbf{s} = \dfrac{2\sqrt{3}\mathbf{h}}{3} = 1{\cdot}155\mathbf{h}$.

Use this formula to find correct to two places of decimal :

10. The side of an equilateral triangle 6 in. high.
11. The side of an equilateral triangle 5 in. high.
12. The side of an equilateral triangle $12x$ ft. high.

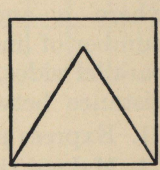
Now solve these miscellaneous problems :

13. If the side of a square is $(x + y)$ in., find : (i) its area, (ii) its perimeter.
14. If a rectangle is $(a + b)$ cm. long and $(c + d)$ cm. broad, what is : (i) its perimeter, (ii) its area ?
15. If the side of a square is 1 in., what is the area of the square on its diagonal ?
16. If the side of a square is $(a + b)$ in., what is the area of the square on its diagonal ?

EXERCISE 73.—SQUARE AND EQUILATERAL TRIANGLE

The diagram shows a square and an equilateral triangle on the same base. Let the base be s units long. The area of the equilateral triangle

$$= \frac{1}{2}sh = \frac{1}{2}s \times \frac{\sqrt{3}}{2}s = \frac{\sqrt{3}}{4}s^2 = 0{\cdot}433s^2.$$



s

Learn: For the **square** . . . **A=s²**. For the **equilateral triangle** . . . **A = 0·433s²**.

Remember that the area of an equilateral triangle is a little less than half the area of a square on the same base. It is about $\frac{1}{8}$ less. This fact will help you to check your results.

Find the area of (i) a square and (ii) an equilateral triangle constructed on each of the lines whose lengths are given below :

1. 3 in. 2. 5 in. 3. 6 in. 4. 20 ft.

5. A piece of wire 12 in. long is bent to form a square. What is the area of the square ? If it had been bent to form an equilateral triangle, what would have been the area of the triangle ?

6. A piece of wire 18 in. long is bent to form a square, and another piece of the same length is bent to form an equilateral triangle. What are the areas of the two figures formed ?

The area of an equilateral triangle in terms of its height is given by this formula :

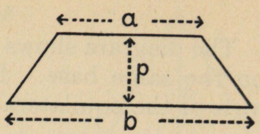
$$A = \frac{\sqrt{3}}{3}h^2 = 0{\cdot}577h^2.$$

Find the areas of the equilateral triangles whose heights are given below :

7. 2 in. 8. 3 in. 9. 2·5 in. 10. 3·2 in.

11. 4·4 cm. 12. 5·3 cm.

EXERCISE 74.—MENSURATION FORMULÆ II

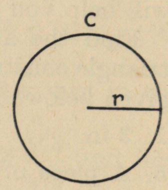


The area of a trapezium is given by this formula :

$$A = \tfrac{1}{2}(a + b)p$$

where a and b represent the number of linear units in the two parallel sides, and p the number in the perpendicular distance between them.

1. Express p in terms of A, a, and b.
2. Make a the subject of the formula.
3. What is the area of the trapezium if $a = 3$ in., $b = 5$ in., and $p = 2{\cdot}4$ in. ?
4. The area of a trapezium is 5·25 sq. in. The two parallel sides are 2·5 and 3·5 in. respectively. What is the perpendicular distance between them ?



In the following formulæ for the area, diameter, radius, and circumference of a circle the A is put as a capital letter because it stands for square units, while the other letters stand for linear units. π is taken as 3·1416.

$$c = \pi d$$

$$c = 2\pi r$$

$$r = \frac{c}{2\pi} = \frac{1}{2\pi}c = 0{\cdot}1592c$$

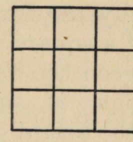
$$\mathbf{A} = \pi \mathbf{r}^2$$

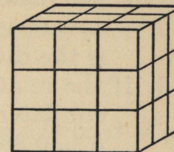
$$\mathbf{A} = \pi\left(\frac{c}{2\pi}\right)^2 = \frac{\pi c^2}{4\pi^2} = \frac{c^2}{4\pi} = \mathbf{0{\cdot}0796c^2}$$

5. What is the circumference of a circle whose radius is 1·5 in. ?
6. A circle has a radius of 2 in. What is its area ?
7. A piece of wire 8 in. long is bent round into a circle. What area will it enclose? What area would it have enclosed if it had been bent to form a square ?

EXERCISE 75.—LENGTHS, AREAS, AND VOLUMES

Line	Square	Cube
1	1	1
2	4	8
3	9	27
4	16	64
5	25	125
6	36	216





Study the above numbers and diagrams.

1. If on a line of 2 in. I construct a square, what is its area ?
2. If on a line of 4 in. I construct a square, what is its area ?
3. If on a line of 6 in. I construct a square, what is its area ?
4. If the line is made twice as large, how many times as large is the area of the square made ?
5. If the line is made three times as large, how many times as large is the area of the square made ?
6. If a rectangle is 3 in. long and 2 in. broad, what is its area ? If I double the length, what is the area then ? If I double both length and breadth, what is the area then ?
7. If with a radius of 2 in. I describe a circle, what is its area ? (Keep π as it is.) If the radius is 4 in., what is the area ? If the radius is 6 in., what is the area ?
8. If I make all the linear measurements of a plane figure twice as large, I make the area . . . times as large. What word is missing ?
9. If I make all the linear measurements of a plane figure three times as large, I make the area . . . times as large. What is the missing word ?
10. If the edge of a cube is 3 in., what is its volume ? If the edge of the cube is 6 in., what is its volume ?
11. If I double the edge of a cube I make the volume of the cube . . . times as great.

1. If the relation between the diagonal and the side of a square is shown by the formula $d = 1{\cdot}414s$, what is the diagonal of a square whose side is 1·5 in. ?
2. If the relation between the height and the side of an equilateral triangle is shown by the formula $h = 0{\cdot}866s$, find the height of an equilateral triangle whose side is 2·5 in.
3. A piece of wire 4·2 in. long is bent to form an equilateral triangle. What area will it enclose ?
4. A piece of wire 7 in. long is bent into a circle. What area will it enclose ?
5. Two rectangular carpets are *similar* in shape, but one is four times as long as the other. The area of the larger carpet is . . . times as great as that of the smaller. What is the missing number ?
6. What must be added to $9a^2 - 9ab + 4b^2$ so that it will factorize into two equal factors ?
7. Solve $(x - 3)(x + 2) = x(x - 1{\cdot}5)$.
8. Solve : $x + 2y = 3$.

   $\frac{x}{4} + 6y = 3\frac{1}{2}$.
9. Factorize $0{\cdot}25p^2 - 0{\cdot}25q^2$.
10. If $\frac{a}{x} = \frac{c}{b}$ what is the value of x ?
11. Change the formula $s = \frac{1}{2}ft^2$ so that t is the subject.
12. If $a = 0{\cdot}6$ and $b = 1{\cdot}4$, what does $2a^2 - 3ab$ equal ?
13. Factorize $64a^3 + 1$.
14. If the hypotenuse of a right-angled triangle is 2·2 in. and one of the other sides is 0·8 in., find within one-tenth of an inch the other side.
15. If $c = \pi d$ and $d = 2r$, express r in terms of c and π.
16. If the perimeter of an equilateral triangle is 3·9 in., what is its height ?
17. Nelson's statue in Trafalgar Square is three times his real height. Assuming it to be made of solid metal, state how many times as heavy it is as it would have been if it had been made life-size.

1. Prove that the relation between the diagonal of a square and its side is given by the formula $d = \sqrt{2}s$, and that consequently $s = \frac{\sqrt{2}d}{2}$.
2. Find to the nearest tenth of an inch the diagonal of a square whose side is 2·4 in. If the diagonal had been 2·4 in., what would the side have been ?
3. Prove that the height of an equilateral triangle is given by the formula $h = \frac{\sqrt{3}}{2}s$, and that its area is given by the formula $A = \frac{\sqrt{3}}{4}s^2$.
4. Three pieces of wire each 10·8 in. long are bent so as to form respectively a circle, a square, and an equilateral triangle. What are the areas enclosed?
5. There are two oval windows exactly similar in shape but different in size. The longest diameter of the smaller window bears to that of the larger the ratio of 2 : 3. What is the ratio of their areas ?
6. There are two statues of the same shape, and their heights are in the ratio of 2 : 5. What is the ratio of their volumes ?
7. Solve $(x - 5)(x + 1) = x(x - 0{\cdot}75)$.
8. Solve : $2x - 3y = 2\frac{1}{2}$.

$$\frac{x}{3} + \frac{y}{4} = \frac{1}{6}.$$

9. Factorize $0{\cdot}04p^2 - 0{\cdot}09q^2$.
10. If $\frac{a}{x+1} = \frac{b}{x-1}$, what is the value of x ?
11. Make s the subject of the formula $A = \pi r(r + s)$.
12. If $x = - 0{\cdot}2$, $y = 1{\cdot}4$, and $z = 0{\cdot}35$, find the value of $x^2 + 2xy - yz + z^2$.
13. If $\frac{1}{9}$ is one of the three factors of $\frac{8x^3}{9} + 3y^3$, what are the other two ?

$$(a + b)^2 = a^2 + 2ab + b^2$$

The square of the sum of two numbers equals the sum of their squares plus twice their product.

$$(a - b)^2 = a^2 - 2ab + b^2$$

The square of the difference of two numbers equals the sum of their squares minus twice their product.

$$(a + b)(a - b) = a^2 - b^2$$

The product of the sum and difference of two numbers equals the difference of their squares.

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$
$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$
$$(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$
$$(a - b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$$
$$x^a \times x^b = x^{a+b}$$
$$x^a \div x^b = x^{a-b}$$

The rule of signs for multiplication : *Like signs give plus ; unlike signs give minus.*

Rule for subtraction : Change all the signs in the expression to be subtracted and then proceed as in algebraic addition.

Rule for brackets : When terms are placed within brackets which have a minus sign in front, all their signs must be changed.

When terms are removed from brackets which have a minus sign in front, all their signs must be changed.

Rules for transposition (i.e. changing sides) in equations : (i) The two sides may be changed bodily without changing signs.

(ii) If any term is transposed from one side to the other its sign must be changed.

The areas of similar plane figures vary as the squares of corresponding linear measurements.

The volumes of similar solids vary as the cubes of corresponding linear measurements.